EFFECTIVE ACTIONS, RADII AND ELECTROMAGNETIC POLARIZABILITIES OF HADRONS IN QCD STRING THEORY

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Abstract

A nonperturbative approach to QCD describing confinement and chiral symmetry breaking is discussed. It is based on the path integral representation of Green's function of quarks and leads to the QCD string theory. The effective actions for mesons and baryons in the external uniform static electromagnetic fields are obtained. The area law of the Wilson loop integral, the approximation of the Nambu-Goto straight-line string, and the asymmetric quark-diquark structure of nucleons are used to simplify the problem. The spin-orbit and spin-spin interactions of quarks are treated as a perturbation. Using the virial theorem we estimate the mean radii of hadrons in terms of the string tension and the Airy function zeros. On the basis of the perturbation theory in small external electromagnetic fields we derive the electromagnetic polarizabilities of nucleons. The electric and diamagnetic polarizabilities of a proton are $\bar{\alpha}_p = 10 \times 10^{-4}\ fm^3$, $\beta_p^{dia} = -8 \times 10^{-4}\ fm^3$ and for a neutron we find $\bar{\alpha}_n = 4.2 \times 10^{-4}\ fm^3$, $\beta_n^{dia} = -5.4 \times 10^{-4}\ fm^3$. Using the Δ contribution to the paramagnetic polarizability of the nucleons, reasonable values of the magnetic polarizabilities $\bar{\beta}_p = (5 \pm 3) \times 10^{-4}\ fm^3$, $\bar{\beta}_n = (7.6 \pm 3) \times 10^{-4}\ fm^3$ are estimated.

1 Introduction

It is now understood that Quantum Chromodynamics (QCD) (see [1]) really describes the strong interactions of hadrons. However, QCD deals with nonobserved objects: quarks and gluons. The standard perturbation theory in the small QCD coupling constant (α_s) uses Feynmann diagrams and is

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applicable in the ultraviolet region. In the infrared region, which is important for hadron physics, the coupling constant is large, and it is impossible to apply perturbation technique. In this case the nonperturbative methods should be used. One of the powerful methods is lattice simulations of QCD giving data about hadron spectrum, matrix elements, and scattering. The second way is to develop analytical nonperturbative methods which allow us to understand deeper nonperturbative phenomena. There are some phenomenological approaches for describing hadron physics such as potential models [2], method of the effective Lagrangians [3], the QCD sum rule approach [4] and others. However, these schemes do not deal with the fundamental QCD Lagrangian.

One of the important problems of particle physics in the infrared region is the confinement of quarks. According to this phenomenon only colourless objects (hadrons) are observable. This means that quarks and gluons which are present in the QCD Lagrangian are absent in the physical spectrum. Forces between quarks and gluons combine them into hadronic colourless states. The appearance of such forces with a linear potential is explained by the string configurations of gluonic fields named the QCD string. Mathematically this phenomenon is expressed by the Wilson area law. Confinement of quarks and gluons can be explained by the dual Meissner effect [5] in the framework of the Abelian-projected method [6]. In accordance with this scenario Abelian magnetic monopoles appear after Abelian projecting Yang-Mills gluonic fields. These magnetic monopoles condense to form vacuum with flux tubes (strings) between quarks. The same picture occurs in the Ginzburg-Landau theory of superconductivity and its relativistic version - Abelian Higgs Model. In these theories the magnetic Abrikosov-Nilsen-Olesen strings [7] appear due to the condensation of Cooper pairs (electrons). Thus, chromoelectric strings which ensure confinement in the framework of QCD are dual to the Abrikosov-Nilsen-Olesen strings.

It should be noted that QCD strings are treated as effective and are real gluonic flux tubes. This is a distinction of QCD strings in $4D$ which can be described by the Nambu-Goto action [8] from the fundamental strings [9]. Therefore there is no problem to quantize QCD strings and we avoid the problem of conformal anomaly in $D \neq 26$. Superstring theories were introduced to unify strong, electroweak, and gravitational interactions.

The goal of this review is to outline the effective string theory which appears from QCD in the infrared region, and allow us to make some estimations of electromagnetic characteristics of hadrons.

There is another nonperturbative phenomenon of QCD - chiral symmetry breaking (CSB) the QCD Lagrangian with massless quarks. For massless quarks, the QCD Lagrangian possesses the chiral $U(N_f) \times U(N_f)$ symmetry which is spontaneously broken. As a result there is no parity degeneracy of hadrons. The real difference of hadron masses with opposite parity is few hundred MeV. The consequence of chiral symmetry breaking QCD is the presence of the Goldstone bosons in the physical spectrum, which are associated with π-mesons.

CSB leads to a nonvanishing quark condensate ($\langle \bar{q}q \rangle$) [10]. As a result the light quarks (u, d quarks) with current masses $m_u \simeq m_d \simeq 7\ MeV$ acquire the dynamical masses $\mu_u \simeq \mu_d \simeq 320\ MeV$. This phenomenon is important for light pseudoscalar mesons because they possess the Nambu-Goldstone nature. Nonzero gluon condensate is responsible for confinement [4]. To obtain low masses of pseudoscalar mesons one needs to take into account the spin interactions of quarks.

Some models were proposed for the explanation of CSB. In instanton vacuum theory (IVT) [11], CSB was explained by the nonvanishing density of quark zero modes. This approach is based on the representation of vacuum as an ensemble of classical instanton- antiinstanton configurations. To have zero topological charge of the QCD vacuum the number of instantons should be equal to the number of antiinstantons. However, a superposition of fields with opposite topological charges is instable as instantons and antiinstantons can annihilate, and in addition such a superposition of fields is not a solution of classical equations of motion. Classical configurations should have high density to reproduce the phenomenological gluon condensate [4], and can distort the original solutions. Besides, IVT can not explain confinement of quarks.

In [12] CSB was confirmed on the basis of gluon propagator which is the solution of Dyson-Schwinger equations.

CSB was also studied in detail in the framework of Nambu-Jona-Lasinio models [13-15] based on four-fermion interactions.

Recently a new analytical method for studying nonperturbative effects of QCD appeared [16-20] called Method of Field Correlators (MFC) which is the generalization of the QCD sum rule approach [4]. MFC is based on QCD and takes into account both gluonic strings and CSB. The fundamental variables of MFC are the gluonic condensate and the correlation length of gluonic fields. This method is based on first principles of QCD and is supported by experiment and lattice data. It can solve problems which can not be solved in the framework of relativistic quark models. Such problems include: (i) the Regge slopes of meson and baryon trajectories and their intercepts, (ii) constituent masses of quarks, (iii) radiuses and electromagnetic polarizabilities of hadrons, (iiii) high energy scattering, and others.

In the framework of MFC both non-perturbative effects of strong interactions (confinement and CSB) can be explained by introducing stochastic gluon vacuum fields with the definite fundamental correlators [16,17]. Then the linear potential between quarks appears and it provides the confinement of quarks. The confinement of quarks does not allow them to be observed, i.e. quarks can not move outside of hadrons at large distances relative each other. This was confirmed by Monte-Carlo simulations and experiments.

The Regge trajectories are asymptotically linear in this approach with a universal slope [17]. There are different stochastic vacuum configurations which are responsible for CSB: instantons (antiinstantons), pieces of (anti-)self-dual fields (for example torons or randomly distributed lumps of field) and others. The necessary requirement is to have zero fermion modes. The condensation of zero modes leads to CSB.

So the MFC in nonperturbative QCD and the dynamics of zero modes give the explication of the double nature of light pseudoscalar particles (pions, kaons and others) as Nambu-Goldstone particles and as the quark-antiquark system with a confining linear potential. It should be noticed that confinement prevents the delocalization of zero modes over the whole volume [18] i.e. stabilizes the phenomena of CSB. The familiar PCAC (partial conservation of axial vector current) theorems and the soft pion technique are reproduced in this approach [19].

There are some difficulties in evaluating meson characteristics in the general case of the complicated string configuration. Naturally, as a first step, we make some approximations and model assumptions to simplify the calculations. So here we consider the straight-line string as a simple configuration and quarks attached to the ends of the string. Such configurations were also studied in [21]. The quark-diquark structure of baryons is employed here.

Spin degrees of freedom are treated as a perturbation and therefore it is questionable to apply this scheme to pions and kaons. For example ρ and K^* mesons can be considered here because the energy shift for them due to the hyperfine spin interaction is below 100 MeV [2]. Here short-range spin-orbital $\mathbf{L}\cdot\mathbf{S}$ and spin-spin $\mathbf{S_1}\cdot\mathbf{S_2}$ interactions are not taken into account. The Coulomb like short-range contribution due to the asymptotic freedom of QCD can be easily included. We imply also that in the presence of light quarks the structure of vacuum yields an area law of the Wilson loop integral. In the present review we investigate hadrons in external, constant, and uniform electromagnetic fields and use the path integral approach.

It is important to calculate different intrinsic characteristics of hadrons on the basis of QCD string theory and to compare them with the experimental values. It will be the test of this scheme. The mean radiuses (and electromagnetic form-factors) and electromagnetic polarizabilities of hadrons are fundamental constants which characterize the complex structure of particles. These values for some hadrons are known from the experimental data and therefore the estimation of them is reasonable.

The electromagnetic polarizabilities of hadrons α, β enter the induced electric $\mathbf{D} = \alpha\mathbf{E}$ and magnetic $\mathbf{M} = \beta\mathbf{H}$ dipole moments, where $\mathbf{E}$, $\mathbf{H}$ are the strengths of electromagnetic fields. As a result there is a contribution to the polarization potential [22] as follows $U(\alpha, \beta) = -(1/2)\alpha E^2 - (1/2)\beta H^2$. Electromagnetic polarizabilities are fundamental low-energy characteristics of strong hadron interactions and therefore they can be calculated in the framework of nonperturbative quantum chromodynamics - QCD string theory.

The review is organized as follows. In Section 2 after describing the general background we derive the Green function of quark-antiquark system (meson). The effective action and Hamiltonian for mesons in external electromagnetic fields is found in Section 3. In Section 4 we estimate the mean size of mesons on the basis of the virial theorem. Section 5 contains the derivation of the Green Function of baryons (three-quark system). The effective action for baryons in external electromagnetic fields based on the proper time method and Feynman path-integrals is derived in Section 6. Section 7 contains the calculation of average distances between quarks in nucleons. The electric polarizabilities of nucleons are evaluated in Section 8. We derive diamagnetic polarizabilities of protons and neutrons using the perturbative expansion in the small magnetic fields in Section 9. In section 10 the method of field correlators and cluster expansion are briefly described. The introduction to the hadron electromagnetic polarizabilities is given by Appendix. In the conclusion we made the comparison of our results with other approaches.

Units are chosen such that $\hbar = c = 1$.

2 Green's Function of Quark-Antiquark System

Here we derive the Green function of quark-antiquark system using the Schwinger proper time method and the Feynman path-integrals. Our goal is to calculate some electromagnetic characteristics of hadrons. For this purpose we need the effective actions for mesons and baryons in external electromagnetic fields. The method of the Green functions will be adopted.

Let us consider the Lorentz and gauge invariant combination of two quark colorless system (meson) in Minkowski space [17]

$$X_M(x,\bar{x},C) = \bar{q}_a(x)\Gamma_A\Phi_{ab}(x,\bar{x})q_b(\bar{x}), \tag{1}$$

where $q_b(\bar{x})$ and $\bar{q}_a(x)$ are quark bispinors, $\bar{q}_a(x) = q_a^+(x)\gamma_4$; $q_a^+(x)$ is the Hermitian-conjugate quark field; $\Gamma_A = 1,\ \gamma_5,\ \gamma_\mu,\ i\gamma_5\gamma_\mu,\ \Sigma_{\mu\nu} = -(i/4)[\gamma_\mu,\gamma_\nu]$ ($\Gamma_A^+ = \Gamma_A$, i.e. Γ_A are Hermitian matrices); γ_μ are the Dirac matrices; a, b are colour indexes and as usual there is a summation on repeating indexes. We imply that quark fields $\bar{q}(x)$, $q(\bar{x})$ possess the definite flavours which form mesons. The gauge invariance is guaranteed here by introducing the parallel transporter [17]:

$$\Phi(x,\bar{x}) = P\exp\left\{ig\int_{\bar{x}}^{x} A_\mu dz_\mu\right\}, \tag{2}$$

where P is the ordering operator along the contour C of integration, g is the coupling constant, $A_\mu = A_\mu^a\lambda^a$; A_μ^a are the gluonic fields; λ^a are the Gell-Mann matrices. Although we have an attractive feature - gauge invariance, the function (1) depends on the form of the contour C. As $X_M(x,\bar{x},C)$ is a gauge invariant object, it obeys the Gauss law on the spacelike surface Σ. The path of integration in Eq. (2) is arbitrary.

Let the four-points x, $\bar{x}$, and y, $\bar{y}$, be the initial and final positions of quark-antiquark, respectively. Two particle quantum Green function is defined as [17]:

$$G(x,\bar{x};y,\bar{y}) = \langle X_M(x,\bar{x},C)X_M^+(y,\bar{y},C')\rangle, \tag{3}$$

were $X_M^+(y,\bar{y},C')$ corresponds to the final state of a meson:

$$X_M^+(y,\bar{y},C') = \bar{q}_a(\bar{y})\Phi_{ab}(\bar{y},y)\bar{\Gamma}_A q_b(y), \tag{4}$$

where $\bar{\Gamma}_A = \gamma_4\Gamma_A\gamma_4$. The brackets $\langle ...\rangle$ mean the path-integrating over gluonic and quark fields:

$$\langle X_M X_M^+\rangle = \int D\bar{q}DqDA_\mu \exp\left\{iS_{QCD}\right\} X_M X_M^+, \tag{5}$$

with the QCD action S_{QCD}. We imply that the measure DA_μ in the path integral (5) includes the well known gauge-fixing and Faddeev-Popov terms for gluonic fields [23]. The Minkowski space is used here but it is not difficult to go into Euclidean space to have well defined path-integrals.

Let us introduce the generating functional for Green's function to calculate the path-integral (5) with respect to quark fields:

$$Z[\bar{\eta},\eta] = \int D\bar{q}Dq \exp\left\{iS_{QCD} + i\int dx\,(\bar{q}_a(x)\eta_a(x) + \bar{\eta}_a(x)q_a(x))\right\}, \quad (6)$$

where we introduce the external colour anticommutative sources (Schwinger sources) η_a, $\bar{\eta}_a$. Then the Green function (3) can be written as

$$G(x,\bar{x};y,\bar{y}) = \int DA_\mu \Big[\frac{\delta}{\delta\eta_a(x)}\Gamma_A\Phi_{ab}(x,\bar{x})\frac{\delta}{\delta\bar{\eta}_b(\bar{x})}$$

$$\times\frac{\delta}{\delta\eta_m(\bar{y})}\Phi_{mn}(\bar{y},y)\bar{\Gamma}_A\frac{\delta}{\delta\bar{\eta}_n(y)}Z[\bar{\eta},\eta]\Big]_{\eta=\bar{\eta}=0}. \quad (7)$$

Now it is possible to integrate the path-integral in Eq.(7) over quark fields $\bar{q}$, q as expression (6) is a Gaussian integral. We may represent the QCD action in the form of

$$S_{QCD} = S(A) - \int dx\bar{q}(x)\left(\gamma_\mu D_\mu + m\right)q(x), \quad (8)$$

where $S(A)$ is an action for gluonic fields with the included ghost fields, $D_\mu = \partial_\mu - iQA^{el}_\mu - igA_\mu$; A^{el}_μ is the vector potential of an electromagnetic field, Q is the charge matrix of quarks, $Q = diag(e_1, e_2, ...e_{N_f})$, e_i are charges of quarks, N_f is the number of flavours; m is the quark mass matrix, $m = diag(m_1, m_2, ...m_{N_f})$ and we imply the summation on colour and flavour indexes. Thus we have just introduced the interaction of quarks with an electromagnetic field. Inserting Eq. (8) into Eq. (6) and integrating with respect to quark fields, we arrive at the expression

$$Z[\bar{\eta},\eta] = \det(-\gamma_\mu D_\mu - m)\exp\left\{iS(A) + i\int dxdy\bar{\eta}(x)S(x,y)\eta(y)\right\}, \quad (9)$$

where the classical quark Green function $S(x,y)$ is the solution of the equation

$$\left(\gamma_\mu D_\mu + m\right)S(x,y) = \delta(x-y). \quad (10)$$

Using Eq. (9) and calculating the variation derivatives in Eq. (7) we find the quantum Green function of a meson:

$$G(x,\bar{x};y,\bar{y}) = \int DA_\mu \det(-\gamma_\mu D_\mu - m)\exp\{iS(A)\}$$
$$\times\Big(tr\Big[\bar{\Gamma}_A S_{na}(y,x)\Gamma_A \Phi_{ab}(x,\bar{x})S_{bm}(\bar{x},\bar{y})\Phi_{mn}(\bar{y},y)\Big]$$
$$-tr\Big[\Phi_{mn}(\bar{y},y)\bar{\Gamma}_A S_{nm}(y,\bar{y})\Big]tr\Big[\Gamma_A\Phi_{ab}(x,\bar{x})S_{ba}(\bar{x},x)\Big]\Big). \qquad (11)$$

The second term in (11) corresponds to the annihilation graphs and it does not contribute to the nonsinglet channel. The functional determinant in Eq. (9) describes the contribution from the vacuum polarization and gives the additional quark loops. In the quenched approximation the fermion determinant is taken to be unity. Neglecting quark-antiquark vacuum loops (quenched approximation) and omitting the annihilation graph, the Green function of a meson (the quark-antiquark system) takes the form (see also [17])

$$G(x,\bar{x};y,\bar{y}) = \langle tr[\bar{\Gamma}_A S_{na}(y,x)\Gamma_A\Phi_{ac}(x,\bar{x})S_{cm}(\bar{x},\bar{y})\Phi_{mn}(\bar{y},y)]\rangle_A, \qquad (12)$$

were the brackets $\langle ...\rangle_A$ mean the averaging over the external vacuum gluonic fields with the standard measure $\exp\left[iS\left(A\right)\right]$.

Let us derive the classical one-quark Green function which is the solution of Eq. (10) using the Fock-Schwinger method. Starting with the approach [24], the solution to Eq. (10) for the Green function of a quark (in Minkowski space) is given by

$$S(x,y) = \langle x \mid (\widehat{D}+m_1)^{-1} \mid y\rangle = \langle x \mid (m_1 - \widehat{D})(m_1^2 - \widehat{D}^2)^{-1} \mid y\rangle$$
$$= \langle q(x)\bar{q}(y)\rangle, \qquad (13)$$

where e_1 and m_1 are the charge and current mass of the quark, $\widehat{D} = \gamma_\mu D_\mu$, $D_\mu = \partial_\mu - ie_1 A_\mu^{el} - igA_\mu$, $A_\mu = A_\mu^a\lambda^a$; γ_μ, λ^a are the Dirac and Gell-Mann matrices, respectively; A_μ^{el}, A_μ^a are the electromagnetic and gluonic vector potentials. The inverse operator $\left(m_1^2 - \widehat{D}^2\right)^{-1}$ can be represented in the proper time s [24]:

$$\left(m_1^2 - \widehat{D}^2\right)^{-1} = i\int_0^\infty ds \exp\left\{-is\left(m_1^2 - \widehat{D}^2\right)\right\}. \qquad (14)$$

Using the properties of the Dirac matrices $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ we find the squared operator

$$\widehat{D}^2 = D_\mu^2 + \Sigma_{\mu\nu}\left(e_1 F_{\mu\nu}^{el} + gF_{\mu\nu}\right), \tag{15}$$

where

$$\Sigma_{\mu\nu} = -\frac{i}{4}[\gamma_\mu, \gamma_\nu], \qquad F_{\mu\nu}^{el} = \partial_\mu A_\nu^{el} - \partial_\nu A_\mu^{el},$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \tag{16}$$

$\Sigma_{\mu\nu}$ are the spin matrices, and $F_{\mu\nu}^{el}$ and $F_{\mu\nu}$ are the strength of electromagnetic and gluonic fields, respectively. Inserting relationship (14) into Eq. (13) with the help of Eq. (15) we get

$$S(x,y) = i\int_0^\infty ds\langle x \mid (m_1 - \widehat{D})$$
$$\times \exp\{-is[m_1^2 - D_\mu^2 - \Sigma_{\mu\nu}(e_1 F_{\mu\nu}^{el} + gF_{\mu\nu})]\} \mid y\rangle. \tag{17}$$

The exponent in Eq. (17) plays the role of the evolution operator which defines the dynamics with the "Hamiltonian" $m_1^2 - D_\mu^2 - \Sigma_{\mu\nu}\left(e_1 F_{\mu\nu}^{el} + gF_{\mu\nu}\right)$ with initial $\mid y\rangle$ and final $\langle x \mid$ states where s means the proper time. Therefore it is convenient to represent the matrix element in Eq. (17) as a path integral [25]:

$$S(x,y) = i\int_0^\infty dsN\int_{z(0)=y}^{z(s)=x} DpDzP\left(m_1 - \widehat{D}\right)\exp\Big[i\int_0^s dt\Big[p_\mu \dot{z}_\mu - m_1^2$$
$$-\left(p_\mu - e_1 A_\mu^{el} - gA_\mu\right)^2 + \Sigma_{\mu\nu}\left(e_1 F_{\mu\nu}^{el} + gF_{\mu\nu}\right)\Big]\Big], \tag{18}$$

where $z_\mu(t)$ is the path of a quark with the boundary conditions $z(0) = y$, $z(s) = x$, $\widehat{D} = i\gamma_\mu\left(p_\mu - e_1 A_\mu^{el} - gA_\mu\right)$ and P means ordering; N is a constant which is connected to the measure definition and it will be chosen later. The path integration over the momenta can be rewritten in the form (see [17,26]

$$N\int Dp\left(m_1 - \widehat{D}\right)\exp\left\{i\int_0^s dt\left[p_\mu \dot{z}_\mu - \left(p_\mu - e_1 A_\mu^{el} - gA_\mu\right)^2\right]\right\}$$
$$= N\int Dp\exp\left[i\int_0^s dt\,(p_\mu \dot{z}_\mu)\right]\left(m_1 + \frac{1}{2}\gamma_\mu \frac{\delta}{\delta p_\mu}\right)$$
$$\times\exp\left\{-i\int_0^s dt\left(p_\mu - e_1 A_\mu^{el} - gA_\mu\right)^2\right\}$$

$$= N\int Dp\left(m_1 - \frac{i}{2}\gamma_\mu \dot{z}_\mu\right)\exp\left\{i\int_0^s dt\left[p_\mu \dot{z}_\mu - \left(p_\mu - e_1 A^{el}_\mu - gA_\mu\right)^2\right]\right\}$$

$$= N\int Dp\left(m_1 - \frac{i}{2}\gamma_\mu \dot{z}_\mu\right)\exp\left\{i\int_0^s dt\left[-p^2_\mu + \frac{1}{4}\dot{z}^2_\mu + \left(e_1 A^{el}_\mu + gA_\mu\right)\dot{z}_\mu\right]\right\}. \tag{19}$$

In Eq. (19) we used the integration by parts (see [26]) and made a continuity of shifts $p_\mu \to p_\mu + e_1 A^{el}_\mu + gA_\mu$ and then $p_\mu \to p_\mu + \dot{z}_\mu/2$. The constant N in Eq. (19) is defined by the relation

$$N\int Dp\exp\left\{-i\int_0^s dt\left(p^2_\mu\right)\right\} = 1. \tag{20}$$

Taking into account Eqs. (19), (20) we find from Eq. (18) the Green function of the quark:

$$S(x,y) = i\int_0^\infty ds\int_{z(0)=y}^{z(s)=x} Dz\ \left(m_1 - \frac{i}{2}\gamma_\mu \dot{z}_\mu(t)\right)P\exp\left\{i\int_0^s dt\Big[\frac{1}{4}\dot{z}^2_\mu(t) - m_1^2\right.$$

$$\left. + \left(e_1 A^{el}_\mu + gA_\mu\right)\dot{z}_\mu(t) + \Sigma_{\mu\nu}\left(e_1 F^{el}_{\mu\nu} + gF_{\mu\nu}\right)\Big]\right\}. \tag{21}$$

3 Effective Action for Mesons

To get the effective action for mesons in the external electromagnetic fields we use the Green function (21) of the quark possessing spin in the Minkowski space which can be represented as

$$S(x,y) = i\int_0^\infty ds\int_{z(0)=y}^{z(s)=x} Dz\ \left(m_1 - \frac{i}{2}\gamma_\mu \dot{z}_\mu(t)\right)P\exp\Big\{i\int_0^s\Big[\frac{1}{4}\dot{z}^2_\mu(t) - m_1^2$$

$$+ e_1\dot{z}_\mu(t)A^{el}_\mu(z) + \Sigma_{\mu\nu}\left(e_1 F^{el}_{\mu\nu} + gF_{\mu\nu}\right)\Big]dt\Big\}\Phi(x,y), \tag{22}$$

where $\Phi(x,y)$ is the path ordered product (2). Inserting Eq. (22) for a quark ($S(x,y)$) and antiquark ($S(\bar{y},\bar{x})$) Green functions into Eq. (12) we find the expression for the Green function of a meson:

$$G(x,\bar{x};y,\bar{y}) = tr\int_0^\infty ds\int_0^\infty d\bar{s}\int_{z(0)=x}^{z(s)=y} Dz\bar{\Gamma}_A\left(m_1 - \frac{i}{2}\gamma_\mu \dot{z}_\mu(t)\right)$$

$$\times\langle P_\Sigma P_A\exp\left\{ig\Sigma_{\mu\nu}\int_0^s F_{\mu\nu}(z)\,dt\right\}\Gamma_A\exp\left\{-ig\Sigma_{\mu\nu}\int_0^{\bar{s}} F_{\mu\nu}(\bar{z})\,d\bar{t}\right\}W(C)\rangle_A$$

$$\times \int_{\bar z(0)=\bar y}^{\bar z(\bar s)=\bar x} D\bar z \left(m_2 - \frac{i}{2}\gamma_\mu \dot{\bar z}_\mu \left(\bar t\right)\right)$$

$$\times \exp\Bigg\{ i\int_0^s \left[\frac{1}{4}\dot z_\mu^2(t) - m_1^2 + e_1 \dot z_\mu(t) A_\mu^{el}(z) + e_1 \Sigma_{\mu\nu} F_{\mu\nu}^{el}\left(z\right)\right] dt$$
$$+ i\int_0^{\bar s} \left[\frac{1}{4}\dot{\bar z}_\mu^2(\bar t) - m_2^2 + e_2 \dot{\bar z}_\mu(\bar t) A_\mu^{el}(\bar z) + e_2 \Sigma_{\mu\nu} F_{\mu\nu}^{el}\left(\bar z\right)\right] d\bar t\Bigg\}, \qquad (23)$$

where P_Σ and P_A are the ordering operators of the spin matrices $\Sigma_{\mu\nu}$ and gluonic fields, respectively; e_1, e_2 are charges and m_1, m_2 are current masses of the quark and antiquark; $z_\mu(t)$, $\bar z_\mu(\bar t)$ are the paths of the quark and antiquark with the boundary conditions $z_\mu(0) = x_\mu$, $z_\mu(s) = y_\mu$, $\bar z_\mu(0) = \bar y_\mu$, $\bar z_\mu(\bar s) = \bar x_\mu$ and $\dot z_\mu(t) = \partial z_\mu/\partial t$. As compared with [17, 27] we added the interaction of charged quarks with the external electromagnetic fields. The gauge - and Lorenz - invariant Wilson loop operator is given by

$$W(C) = \frac{1}{N_C} trP \exp\left\{ig \int_C A_\mu dz_\mu\right\}, \qquad (24)$$

where N_C is the color number, and C is the closed contour of lines $x\bar x$ and $y\bar y$ connected by paths $z(t)$, $\bar z(\bar t)$ of the quark and antiquark. The Wilson operator (24) contains both the perturbative and nonperturbative interactions between quarks via gluonic fields A_μ with the QCD coupling constant g, and it is the amplitude of the process of creation and annihilation of quarks and antiquarks.

Using non-Abelian Stokes theorem [28, 17], the Wilson loop is rewritten as

$$W(C) = \frac{1}{N_C} trP \exp\left\{ig \int_\Sigma F_{\mu\nu}(z, z_0) d\sigma_{\mu\nu}(z)\right\}, \qquad (25)$$

where Σ is an arbitrary surface bounded by the contour C, and $F_{\mu\nu}(z, z_0)$ is given by

$$F_{\mu\nu}(z, z_0) = \Phi(z_0, z) F_{\mu\nu}(z) \Phi(z, z_0). \qquad (26)$$

The coordinate z_0 and path of the transporter $\Phi(z, z_0)$ belongs to the surface Σ, and they are arbitrary [17]. The contour C can be parametrized by the four vector $z_\mu(\xi)$, where two dimensional coordinate $\xi = (\xi_1, \xi_2)$ induces the metric tensor of the surface Σ: $g_{\mu\nu}^{ab}(\xi) = [\partial_a z_\mu(\xi)][\partial_b z_\nu(\xi)]$ $(\partial_a = \partial/\partial\xi_a)$. The infinitesimal element of the surface Σ is given by

$$d\sigma_{\mu\nu}(z) = \epsilon^{ab} g_{\mu\nu}^{ab}(\xi) d^2\xi, \qquad (27)$$

where $\epsilon^{ab} = -\epsilon^{ba}, \epsilon^{12} = 1$. In accordance with the approach in [17], spin interactions can be treated as perturbations, e.g. for ρ and K^* mesons but not for Nambu-Goldstone π- and K-mesons. To construct the expressions in spin interactions we write the relationship [17]

$$\langle \exp \left\{ ig\Sigma_{\mu\nu} \left[\int_0^s F_{\mu\nu}(z)\, dt \right] \right\} W(C) \rangle_A$$

$$= \exp \left\{ \Sigma_{\mu\nu} \left[\int_0^s dt \frac{\delta}{\delta\sigma_{\mu\nu}(t)} \right] \right\} \langle W(C) \rangle_A, \tag{28}$$

where $\delta\sigma_{\mu\nu}(t)$ is the surface around the point $z_\mu(t)$. The zeroth order in spin-orbit and spin-spin interactions corresponds to neglecting the terms $e_i \Sigma_{\mu\nu} F^{el}_{\mu\nu}$, $g\Sigma_{\mu\nu} F_{\mu\nu}$ in (23).

To receive the effective action for mesons from Eq. (23) one needs to estimate the Wilson loop integral (24) which carries very important information about an interaction of quarks.

Let us consider the case when the distance between quarks is greater than the time fluctuations of the gluonic fields ($r \gg T_g$). The Monte-Carlo calculations [29] gave $T_g \simeq 0.2 \div 0.3 fm$. We imply that the characteristic quark relative distance is $r \simeq 1\ fm$. This assumption is confirmed by the lattice data [30] and by the calculation of the quark-antiquark relative coordinate [31].

The average Wilson integral (24) at large distances in accordance with the area law can be represented in the Minkowski space as

$$\langle W(C) \rangle_A = \exp(i\sigma S_{min}), \tag{29}$$

where σ is the string tension and S_{min} is the area of the minimal surface inside of the contour C. This behavior of the Wilson loop integral is the consequence of the nonperturbative interactions of a quark and antiquark, and corresponds to the linear potential. The area law of the Wilson loop is the criterion of confinement [32]. In this case the gluonic field between a quark and antiquark forms a string, and the string tension σ is the energy of a string per unit length. In accordance with the lattice data (see e.g. [17]) $\sigma \simeq 0.2\ GeV^2$. It is impossible to calculate a dimensional parameter σ using the perturbative expansion in the small coupling constant g. There is a dimensional parameter of QCD in the high-energy limit - the scale parameter $\Lambda_{QCD} \simeq 200\ MeV$:

$$\Lambda^2_{QCD} = \mu^2 \exp \left(-\frac{16\pi^2}{((11/3)N_C - (2/3)N_f) g^2(\mu^2)} \right), \tag{30}$$

where μ is a normalization mass. According to the phenomenon of dimensional transmutation, dimensional parameters are proportional to some power of Λ_{QCD}. It is easy to see from Eq. (30) that all coefficients of the expansion of Λ_{QCD} in powers of g^2 are zero. Therefore, σ is proportional to Λ^2_{QCD} and the QCD strings are described by the nonperturbative approach.

In the case of perturbative interactions at small distances (high energy), the Coulomb potential dominates, which does not confine quarks. QCD is an asymptotically free theory and at high energy the coupling constant is a small parameter. In this regime the gauge field is not concentrated in the tube (string) but fills the whole space. Then the asymptotic form of the Wilson loop (for a smooth contour C) for this case is given by the perimeter law:

$$\langle W(C)\rangle_A = \exp\{i\gamma L(C)\}\langle W(C)_{ren}\rangle_A, \tag{31}$$

where the appearance of the finite value $\langle W(C)_{ren}\rangle$ is due to the renormalization; γ is a dimensional constant, and $L(C)$ is the length of the contour ($L(C) = \int_0^1 ds\sqrt{\dot{z}^2_\mu(s)}$, $z_\mu(0) = z_\mu(1)$). When the distance between quarks $r < 0.25\ fm$, the system is in the Coulomb phase and we have the perimeter law behavior of the Wilson averaged integral (31). The confining phase (nonperturbative) occurs at large distances and gives the area law (29). For wide class of contours without self-intersections, the Wilson loop integral is multiplicatively renormalizable [33] and is given by

$$\langle W(C)\rangle_A = \langle W(C)\rangle_{pert}\langle W(C)\rangle_{nonpert}, \tag{32}$$

where the first factor in right-hand side of Eq. (32) describes the perturbative contribution in the Wilson integral (31), and the second describes confinement.

The surface S_{min} in Eq. (29) can be parametrized by the Nambu-Goto form [8]

$$S_{min} = \int_0^T d\tau \int_0^1 d\beta\sqrt{(\dot{w}_\mu w'_\mu)^2 - \dot{w}^2_\mu w'^2_\nu}, \tag{33}$$

where $\dot{w}_\mu = \partial w_\mu/\partial\tau$, $w'_\mu = \partial w_\mu/\partial\beta$.

Using the approximation [17] that the coordinates of the string world surface $w_\mu(\tau,\beta)$ can be taken by straight lines for the minimal surface we write

$$w_\mu(\tau,\beta) = z_\mu(\tau)\beta + \bar{z}_\mu(\tau)(1-\beta), \tag{34}$$

where τ is implied to be the proper time parameter for both trajectories $\tau = (tT)/s = (\bar{t}T)/\bar{s}$. Thus we ignore vibrations of the string.

For uniform static external electromagnetic fields we have the representation of the vector potential through the strength tensor $F^{el}_{\mu\nu}$:

$$A^{cl}_\nu(z) = \frac{1}{2}F^{el}_{\mu\nu}z_\mu, \qquad A^{cl}_\nu(\bar{z}) = \frac{1}{2}F^{el}_{\mu\nu}\bar{z}_\mu. \tag{35}$$

The paths z_μ, $\bar{z}_\mu$ are expressed via the center of mass coordinate R_μ and the relative coordinate r_μ [17],

$$\bar{z}_\mu(\tau) = R_\mu - \frac{\bar{s}}{s+\bar{s}}r_\mu, \qquad z_\mu(\tau) = R_\mu + \frac{s}{s+\bar{s}}r_\mu \tag{36}$$

with the boundary conditions for $R_\mu(\tau)$, $r_\mu(\tau)$:

$$R_\mu(0) = \frac{\mu_1 y_\mu + \mu_2 \bar{y}_\mu}{\mu_1+\mu_2}, \quad R_\mu(T) = \frac{\mu_1 x_\mu + \mu_2 \bar{x}_\mu}{\mu_1+\mu_2},$$

$$r_\mu(0) = y_\mu - \bar{y}_\mu, \quad r_\mu(T) = x_\mu - \bar{x}_\mu. \tag{37}$$

The integration with respect to z_μ, $\bar{z}_\mu$ in (23) is replaced by the integration over new variables R_μ, r_μ. As τ is a common time for the quark and antiquark (the time of the meson) the parametrization $z_\mu = (\mathbf{z}, i\tau)$, $\bar{z}_\mu = (\bar{\mathbf{z}}, i\tau)$ is possible [17]. This leads to the constraints: $R_0(\tau) = \tau$, $r_0(\tau) = 0$. In accordance with the approach [17] we introduce the dynamical masses μ_1, μ_2 by relationships

$$\mu_1 = \frac{T}{2s}, \qquad \mu_2 = \frac{T}{2\bar{s}}. \tag{38}$$

Replacing the integration with respect to s, $\bar{s}$ in (23) by the integration over $d\mu_1$ and $d\mu_2$ with the help of Eqs. (29), (33) - (36) we find [31] the two-point function in the zeroth order in spin interactions

$$G(x,\bar{x};y,\bar{y}) = T^2\int_0^\infty \frac{d\mu_1}{2\mu_1^2}\int_0^\infty \frac{d\mu_2}{2\mu_2^2}\int DRDr\; exp\left\{iS_{eff}\right\} \tag{39}$$

with the effective action

$$S_{eff} = \int_0^T d\tau\Big[-\frac{m_1^2}{2\mu_1} - \frac{m_2^2}{2\mu_2} + \frac{1}{2}\left(\mu_1+\mu_2\right)\dot{R}_\mu^2 + \frac{1}{2}\tilde{\mu}\dot{r}_\mu^2$$
$$+\frac{1}{2}F^{el}_{\nu\mu}e\left(\dot{R}_\mu R_\nu + \frac{1}{4}\dot{r}_\mu r_\nu\right) - \frac{q}{4}F^{el}_{\nu\mu}\left(\dot{R}_\mu r_\nu + \dot{r}_\mu R_\nu\right)$$
$$-\int_0^1 d\beta\sigma_0\sqrt{(\dot{w}_\mu w'_\mu)^2 - \dot{w}_\mu^2 w'^2_\nu}\Big], \tag{40}$$

where $w_\mu = R_\mu + [\beta - \mu_1/(\mu_1+\mu_2)]r_\mu$, $\tilde{\mu} = \mu_1\mu_2/(\mu_1+\mu_2)$ is the reduced mass of the quark-antiquark system, $e = e_1 + e_2$, $q = e_1 - e_2$. As a first step we are interested here in the spinless part and therefore the preexponential terms $\left(m_1 - \frac{i}{2}\gamma_\mu \dot{z}_\mu\,(t)\right)$, $\left(m_2 - \frac{i}{2}\gamma_\mu\;\dot{\bar{z}}_\mu\,(\bar{t})\right)$ and the constant matrices $\Sigma_{\mu\nu}F^{el}_{\mu\nu}$ were omitted in Eq. (39). The expression (40) defines the effective Lagrangian for mesons in external uniform static electromagnetic fields in accordance with the formula $S_{eff} = \int_0^T \mathcal{L}_{eff} d\tau$. The expression (40) looks like nonrelativistic one at $F_{\mu\nu} = 0$, but it is not. The author of [17] showed that the relativism is contained here due to the $\tilde{\mu}$ dependence and the spectrum is similar to that of the relativistic quark model.

The mass of the lowest states can be found on the basis of the relationship [34]

$$\int DRDr\exp\left\{iS_{eff}\right\} =$$

$$\langle R = \frac{\mu_1 x + \mu_2\bar{x}}{\mu_1+\mu_2}, r = x - \bar{x} \mid \exp\{-iT\mathcal{M}(\mu_1,\mu_2)\} \mid R = \frac{\mu_1 y + \mu_2\bar{y}}{\mu_1+\mu_2}, r = y - \bar{y}\rangle, \tag{41}$$

where the mass $\mathcal{M}(\mu_1,\mu_2)$ is the eigenfunction of the Hamiltonian. After that the Green function (39) is derived by integrating (40) over the dynamical masses μ_1, μ_2. In accordance with [17] we estimate the last integration on $d\mu_1$, $d\mu_2$ using the steepest descent method which gives a good accuracy when the Minkowski time $T \to \infty$. To have the correct formulas, it is necessary to go into Euclidean space and return into Minkowski space on completing the functional integration. We use this procedure.

The last term in (40) can be represented by the relation

$$\sigma\int_0^1 d\beta\sqrt{(\dot{w}_\mu w'_\mu)^2 - \dot{w}^2_\mu w'^2_\nu}$$

$$= \sigma\int_0^1 d\beta\sqrt{b_0 + 2b_1\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right) + b_2\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right)^2}, \tag{42}$$

where

$$b_0 = \left(r_\mu \dot{R}_\mu\right)^2 - r^2_\mu \dot{R}^2_\nu, \quad b_1 = (r_\mu \dot{r}_\mu)\left(r_\nu \dot{R}_\nu\right) - r^2_\mu\left(\dot{r}_\nu \dot{R}_\nu\right), \quad b_2 = (r_\mu \dot{r}_\mu)^2 - r^2_\mu \dot{r}^2_\nu.$$

In the pure potential regime at low orbital excitations of the string when the orbital quantum number l is small, expression (42) is equal to $\sigma\sqrt{\mathbf{r}^2}$. As the equalities $R_0(\tau) = \tau$, $r_0(\tau) = 0$ are valid, the dynamical quantities are

3-dimensional. From Eqs. (40), (42) using the standard procedure we find the canonical three-momenta corresponding to the center of mass coordinate R_μ and the relative coordinate r_μ:

$$\Pi_k = \frac{\partial \mathcal{L}_{eff}}{\partial \dot{R}_k} = (\mu_1 + \mu_2)\dot{R}_k + \frac{e}{2}F^{el}_{\nu k}R_\nu + \frac{q}{4}F^{el}_{\nu k}r_\nu$$

$$-\sigma \int_0^1 d\beta \frac{\left(r_\mu \dot{R}_\mu\right) r_k - r_\mu^2 \dot{R}_k + \left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right)\left[\left(r_\mu \dot{r}_\mu\right) r_k - r_\mu^2 \dot{r}_k\right]}{\sqrt{b_0 + 2b_1\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right) + b_2\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right)^2}},$$

$$p = \frac{\partial \mathcal{L}_{eff}}{\partial \dot{r}_k} = \tilde{\mu}\dot{r}_k + \frac{e}{8}F^{el}_{\nu k}r_\nu + \frac{q}{4}F^{el}_{\nu k}R_\nu$$

$$-\sigma \int_0^1 d\beta \frac{\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right)\left[\left(r_\mu \dot{R}_\mu\right) r_k - r_\mu^2 \dot{R}_k\right] + \left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right)^2\left[\left(r_\mu \dot{r}_\mu\right) r_k - r_\mu^2 \dot{r}_k\right]}{\sqrt{b_0 + 2b_1\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right) + b_2\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right)^2}}, \tag{43}$$

The Hamiltonian $\mathcal{H} = \pi_k \dot{r}_k + \Pi_k \dot{R}_k - \mathcal{L}_{eff}$ found from (40) with the help of (42), (43) takes the form

$$\mathcal{H} = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_1 + \mu_2}{2} + \frac{\mu_1 + \mu_2}{2}\dot{\mathbf{R}}^2 + \frac{\tilde{\mu}}{2}\dot{\mathbf{r}}^2 - \frac{e}{2}(\mathbf{E}\mathbf{R}) - \frac{q}{4}(\mathbf{E}\mathbf{r})$$
$$+\sigma \int_0^1 d\beta \frac{\mathbf{r}^2}{\sqrt{b_0 + 2b_1\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right) + b_2\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right)^2}}, \tag{44}$$

so that the equation for the eigenvalues is given by

$$\mathcal{H}\Phi = \mathcal{M}(\mu_1, \mu_2)\Phi. \tag{45}$$

As the canonical momentum $\mathbf{\Pi}$ corresponding to the center of mass coordinate is a constant due to the conservation law, i.e. $\mathbf{\Pi} = const$, we can put $\dot{R}_k = 0$, $\dot{R}_4 = i$. At this choice parameters occurring Eq. (44) become (at $r_4 = 0$) $b_0 = \mathbf{r}^2$, $b_1 = 0$, $b_2 = -(\mathbf{r} \times \dot{\mathbf{r}})^2$. Then the last term in Eq. (44) giving the contribution to the energy due to the string is given by

$$\mathcal{H}_{string} = \sigma \int_0^1 d\beta \frac{\mathbf{r}^2}{\sqrt{\mathbf{r}^2 - (\mathbf{r} \times \dot{\mathbf{r}})^2\left(\beta - \frac{\mu_1}{\mu_1+\mu_2}\right)^2}},$$

The expression $\mathcal{H}_{string}$ makes sense here when the value under the root squared is positive. This condition is realized at $(\mathbf{r}\times\dot{\mathbf{r}})^2 < 2\mathbf{r}^2$ (if $\mu_1=\mu_2$), i.e. for low orbital quantum numbers l. After the integration over the parameter β we get

$$\mathcal{H}_{string} = \frac{\sigma\mathbf{r}^2}{\sqrt{(\mathbf{r}\times\dot{\mathbf{r}})^2}}\Bigg[\arcsin\left(\frac{\mu_1}{\mu_1+\mu_2}\sqrt{\frac{(\mathbf{r}\times\dot{\mathbf{r}})^2}{\mathbf{r}^2}}\right) + \arcsin\left(\frac{\mu_2}{\mu_1+\mu_2}\sqrt{\frac{(\mathbf{r}\times\dot{\mathbf{r}})^2}{\mathbf{r}^2}}\right)\Bigg]. \quad (46)$$

In the potential regime at low orbital quantum number when $(\mathbf{r}\times\dot{\mathbf{r}})^2 \ll \mathbf{r}^2$ Eq. (46) is replaced by

$$\mathcal{H}_{string} = \sigma_0\sqrt{\mathbf{r}^2}$$

and it describes the linear potential which guarantees confinement of quarks.

The terms contained the strength of the electric field in Eq. (44) describe the interaction of the dipole electric moment $\mathbf{d}$ with an external electric field. Using the definitions we have

$$\frac{e}{2}(\mathbf{E}\mathbf{R}) + \frac{q}{4}(\mathbf{E}\mathbf{r}) = \frac{1}{2}(e_1\mathbf{r_1} + e_2\mathbf{r_2})\mathbf{E} = \mathbf{d}\mathbf{E} \quad (47)$$

and the interaction energy of the electric dipole moment with a uniform static electric field is $U=-\mathbf{d}\mathbf{E}$.

In the center of mass system when $\mathbf{R}=const$ and at $l=0$, $\mathbf{E}=0$, $\mathbf{H}=0$, taking into account Eq. (43), we find (see [17]) from Eq. (44) the expression

$$\mathcal{H}_0 = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_1+\mu_2}{2} + \frac{\mathbf{p}^2}{2\tilde{\mu}} + \sigma\sqrt{\mathbf{r}^2}. \quad (48)$$

The Hamiltonian (48) describes two quarks which are connected by the non-rotating string. Finding extremum of $\mathcal{H}_0$ in m_1 and m_2: $\partial\mathcal{H}_0/\partial\mu_1 = 0$, $\partial\mathcal{H}_0/\partial\mu_2 = 0$, one arrives at

$$\mu_{10} = \sqrt{\mathbf{p}^2+m_1^2}, \quad \mu_{20} = \sqrt{\mathbf{p}^2+m_2^2},$$

$$\mathcal{H}_1 = \sqrt{\mathbf{p}^2+m_1^2} + \sqrt{\mathbf{p}^2+m_2^2} + \sigma\sqrt{\mathbf{r}^2}. \quad (49)$$

Thus we come to the Hamiltonian of the relativistic quark model (RQM) (see [35]). In the approach considered μ_1 and μ_2 are dynamical masses and m_1,

m_2 are current masses of quarks. In RQM Eq. (49) is used for dynamical masses m_1, m_2 of the order of 200 MeV and arbitrary orbital number l. So the Hamiltonian (49) ignores the rotation of the string and gives the slope of the Regge trajectories $1/8\sigma$ [17]. Besides, in RQM to get the mass of ρ-meson one needs to subtract $700 \div 800$ MeV. Such large negative constant introduced by hand in RQM has nonperturbative nature. The appearance of this constant is due to selfenergies of quarks and can be explained in the framework of CSB in QCD string approach [17].

At large orbital quantum number l the Hamiltonian becomes [17]

$$\mathcal{H}_2^2 = 2\pi\sigma\sqrt{l(l+1)},$$

so that the Regge slope is equal to $1/2\pi\sigma$.

4 Mean size of Mesons

Now we find the wave function of the ground state and estimate the mean size of mesons. The equation for the eigenfunction Φ_0 of the auxiliary "Hamiltonian"

$$\tilde{\mathcal{H}}_0 = \mathcal{H}_0 - \frac{m_1^2}{2\mu_1} - \frac{m_2^2}{2\mu_2} - \frac{(\mu_1 + \mu_2)}{2}$$

is given by

$$\left(\frac{1}{2\tilde{\mu}}\mathbf{p}^2 + \sigma_0\sqrt{\mathbf{r}^2}\right)\Phi_0 = \epsilon(\mu)\Phi_0, \tag{50}$$

so that $\tilde{\mathcal{H}}_0\Phi_0 = \epsilon(\mu)\Phi_0$, where $\epsilon(\mu)$ is the eigenvalue. We can apply equation (50) to the leading trajectories with light quarks with masses $m_1 = m_2 \equiv m$, $\mu_1 = \mu_2 \equiv \mu$ ($\tilde{\mu} = \mu/2$) for ρ mesons and when $m_1 \neq m_2$, $\mu_1 \neq \mu_2$ for K^* mesons [36]. In quantum theory instead of the path integration in $\mathbf{r}$ we can use the replacement $p_k \rightarrow -i\partial/\partial r_k$. Then Eq. (50) becomes

$$\left(-\frac{1}{2\tilde{\mu}}\frac{\partial^2}{\partial r_i^2} + \sigma_0\sqrt{\mathbf{r}^2}\right)\Phi_0 = \epsilon(\mu)\Phi_0, \tag{51}$$

Equation (51) gives the discrete values of the energy $\epsilon(\mu)$ due to the shape of the potential energy. The numerical solution of Eq. (51) was obtained in [40]. It is useful to find the solution to equation (51) for the ground state in analytical form. After introducing the variables $\rho_k = (2\tilde{\mu}\sigma_0)^{1/3} r_k$, $\epsilon(\tilde{\mu}) = (2\tilde{\mu})^{-1/3}\sigma_0^{2/3} a(n)$ [17], Eq. (51) becomes

$$\left(-\frac{\partial^2}{\partial \rho_i^2} + \rho\right)\Phi_0(\rho) = a(n)\Phi_0(\rho), \tag{52}$$

where $\rho = \sqrt{\rho_1^2 + \rho_2^2 + \rho_3^2}$. The solution to Eq. (52) may be chosen in the form $\Phi_0(\rho) = R(\rho)Y_{lm}(\theta, \phi)$, where $Y_{lm}(\theta, \phi)$ are spherical functions. After setting the variable $R(\rho) = \chi(\rho)/\rho$ we come to the equation for the radial function

$$\chi^{''}(\rho) + \left(a(n) - \rho - \frac{l(l+1)}{\rho^2}\right)\chi(\rho) = 0, \tag{53}$$

where $\chi^{''}(\rho) = \partial^2\chi(\rho)/\partial\rho^2$, and l is an orbital quantum number. The solutions to Eq. (53) for the ground state $l = 0$ are the Airy functions $Ai(\rho - a(n))$, $Bi(\rho - a(n))$ [41]. The finite solution to Eq. (53) at $\rho \to \infty$ $(l = 0)$ is

$$\chi(\rho) = NAi(\rho - a(n)). \tag{54}$$

The constant N can be found from the normalization condition

$$\int_0^\infty \chi^2(\rho)d\rho = 1. \tag{55}$$

The requirement that this solution satisfies the condition $\chi(0) = NAi(-a(n)) = 0$ gives the Airy function zeroes [41] $a(1) \equiv a_1 = 2.3381$, $a(2) \equiv a_2 = 4.0879$ and so on. The main quantum number $n = n_r + l + 1$, where n_r is the radial quantum number which defines the number of zeroes of the function $\chi(\rho)$ at $\rho > 0$. For the ground state we should take the solution (54) at $a(n) = a_1$ (here $n_r = 0$, $l = 0$, $n = 1$):

$$\chi_0(\rho) = N_0 Ai(\rho - a_1). \tag{56}$$

For the excited states it is necessary to choose the corresponding value of $n = n_r + l + 1$.

Now let us estimate the mean-squared radius for the state which is described by the function Φ_0 (the solution of Eq. (50)). Multiplying Eq. (50) by the conjugated function Φ_0^* and integrating over the volume we find the relations

$$\langle T\rangle + \langle U\rangle = \epsilon(\tilde{\mu}),$$

$$\langle T\rangle = -\frac{1}{2\tilde{\mu}}\int \Phi_0^*\partial_k^2\Phi_0 dV, \quad \langle U\rangle = \sigma_0\int\sqrt{\mathbf{r}^2}\Phi_0^*\Phi_0 dV. \tag{57}$$

It is seen from Eqs. (57) that the mean potential energy $\langle U\rangle = \sigma_0\left\langle\sqrt{\mathbf{r}^2}\right\rangle$ is connected with the mean diameter $\left\langle\sqrt{\mathbf{r}^2}\right\rangle$ (because $\mathbf{r}$ is the relative coordinate and quarks move around their center of mass), which defines the size of mesons. In accordance with the virial theorem [42] we have the connection of the mean kinetic energy with the mean potential energy:

$$2\langle T\rangle = k\langle U\rangle, \tag{58}$$

where k is defined from the equality $U(\lambda r) = \lambda^k U(r)$. In our case of the linear potential $k = 1$ and from Eqs. (57), (58) we get

$$\langle U\rangle = \frac{2}{3}\epsilon(\tilde{\mu}) = \frac{2}{3}(2\tilde{\mu})^{-1/3}\sigma_0^{2/3}a(n). \tag{59}$$

The use of the steepest descent method for the estimation of the integration in μ (at $\mathbf{H} = 0$) leads to the conditions [17]:

$$\frac{\partial\mathcal{M}(\mu_1,\mu_2)}{\partial\mu_1} = 0, \quad \frac{\partial\mathcal{M}(\mu_1,\mu_2)}{\partial\mu_2} = 0, \tag{60}$$

where the mass of the ground state $\mathcal{M}(\mu_1,\mu_2)$ is given by [see Eqs. (44), (45)]

$$\mathcal{M}(\mu_1,\mu_2) = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_1+\mu_2}{2} + (2\tilde{\mu})^{-1/3}\sigma_0^{2/3}a(n). \tag{61}$$

Here we consider the more general case as compared with [17] when $\mu_1 \neq \mu_2$ ($m_1 \neq m_2$). This case is realized for K^* mesons. It is assumed that the current mass of u,d-quarks ($m_u = 5.6 \pm 1.1\ MeV$, $m_d = 9.9 \pm 1.1\ MeV$ [43]), m_1 is much less than the dynamical mass μ_1 ($\mu_1 \simeq 330\ MeV$), i.e. $m_1 \ll \mu_1$ and the mass of s-quark m_2 ($m_s = 199 \pm 33\ MeV$ [43]) is comparable with μ_1 but $m_2 < \mu_1$. Using these assumptions we neglect the term $m_1^2/2\mu_1$ in Eq. (61) and from Eqs. (60) have the equations

$$(2\tilde{\mu}\sigma_0)^{2/3}a(n) = 3\mu_1^2, \quad 3m_2^2 + (2\tilde{\mu}\sigma_0)^{2/3}a(n) = 3\mu_2^2. \tag{62}$$

From Eqs. (62) we arrive to the expression for the dynamical mass μ_2 (for s-quark):

$$\mu_2 = \sqrt{\mu_1^2 + m_2^2}. \tag{63}$$

To find μ_1 the perturbation in the parameter m_2^2/μ_1^2 will be assumed. Using the relation $\mu_2 \simeq \mu_1(1+m_2^2/(2\mu_1^2))$ which is obtained from Eq. (63) and the definition of the reduced mass $\tilde{\mu} = \mu_1\mu_2/(\mu_1+\mu_2)$ from Eqs. (62) we arrive at the equation

$$\mu_1 \simeq \sqrt{\sigma_0}\left(\frac{a(n)}{3}\right)^{3/4}\left(1+\frac{m_2^2}{8\mu_1^2}\right). \tag{64}$$

In zeroth order we come to the value $\mu_0 \equiv \mu_1^{(0)} = \sqrt{\sigma_0}(a(n)/3)^{3/4}$ [17]. The next order gives the relationship

$$\mu_1 \simeq \sqrt{\sigma_0}\left(\frac{a(n)}{3}\right)^{3/4} + \frac{m_2^2}{8\sqrt{\sigma_0}}\left(\frac{3}{a(n)}\right)^{3/4}. \tag{65}$$

In a particular case $m_2 = 0$ we arrive at $\mu_1 = \mu_2 = \mu_0 = \sqrt{\sigma_0}(a(n)/3)^{3/4}$ [17]. The value of the string tension $\sigma_0 = 0.15\ GeV^2$ was found from a comparison of the experimental slope of the linear Regge trajectories, $\alpha' = 0.85\ GeV^{-2}$, and the variable $\alpha' = 1/8\sigma_0$ [17]. It leads for the lowest state $n_r = 0, l = 0, a(1) = 2.3381$ to the value $\mu_0 = 321\ MeV$ [17]. This means that for ρ-mesons when $m_1 = m_u$, $m_2 = m_d$ we have the dynamical masses of u, d-quarks $\mu_1 = \mu_2 = \mu_0$. For K^*-mesons using Eq. (63) and $m_2 = m_s \simeq 200\ MeV$ [43] from Eq. (65) we get the reasonable values

$$\mu_1 \simeq 337\ MeV, \quad \mu_2 \simeq 392\ MeV, \quad \tilde{\mu} \simeq 181\ MeV. \tag{66}$$

Inserting the equation $\langle U\rangle = \left\langle \sigma_0\sqrt{\mathbf{r}^2}\right\rangle$ into the left-hand side of Eq. (59) produces the expression

$$\left\langle\sqrt{\mathbf{r}^2}\right\rangle = \frac{2}{3}(2\tilde{\mu}\sigma_0)^{-1/3}a(n). \tag{67}$$

From Eqs. (63), (65) using the first order in the parameter m_2^2/μ_1^2 we find

$$2\tilde{\mu} \simeq \mu_0\left(1+\frac{3m_2^2}{8\mu_0^2}\right) \qquad \left(\mu_0 = \sqrt{\sigma_0}\left(\frac{a(n)}{3}\right)^{3/4}\right). \tag{68}$$

Eq. (67) with the help of Eq. (68) gives the approximate relation for the mean relative coordinate

$$\left\langle\sqrt{\mathbf{r}^2}\right\rangle = \frac{2}{\sqrt{\sigma_0}}\left(\frac{a(n)}{3}\right)^{3/4}\left[1+\frac{3m_2^2}{8\sigma_0}\left(\frac{3}{a(n)}\right)^{3/2}\right]^{-1/3}. \tag{69}$$

For ρ-meson putting $m_2 = 0$ in Eq. (69) we arrive at

$$\left\langle \sqrt{\mathbf{r}^2} \right\rangle = \frac{2}{\sqrt{\sigma_0}} \left(\frac{a(n)}{3} \right)^{3/4} . \tag{70}$$

The same expression (70) was found in [31] using another method. With the help of the definition of the center of mass coordinate we can write the approximate relation for the mean charge radius of ρ-mesons [2]:

$$\sqrt{\left\langle r_\rho^2 \right\rangle} \simeq \frac{1}{2} \left\langle \sqrt{\mathbf{r}^2} \right\rangle . \tag{71}$$

At $\sigma_0 = 0.15\ GeV^2$ [17,44] and $a(1) = 2.3381$ Eqs. (70), (71) give

$$\sqrt{\left\langle r_\rho^2 \right\rangle} \simeq 0.42\ fm \quad \Big(\left\langle \sqrt{\mathbf{r}^2} \right\rangle = 0.84\ fm \Big) . \tag{72}$$

The value (72) characterizes the radius of the sphere where the wave function of the ρ meson is concentrated (remember that $\mathbf{r}$ is the distance between quarks). We know only the experimental data for $\pi^\pm$ mesons which have the same quark structure as $\rho^\pm$ mesons [45]:

$$\left\langle r_{\pi^\pm}^2 \right\rangle_{exp} = (0.44 \pm 0.02)\ fm^2 \quad \left(\sqrt{\langle r_{\pi^\pm}^2 \rangle}_{exp} \simeq 0.66\ fm \right) .$$

For calculating the relative coordinate of quarks for K^* mesons we should use Eq. (67) or (69) with the conditions $\mu_1 = \mu_u$ and $\mu_2 = \mu_s$ (Eqs. (66)). As a result formula (67) gives the value of the mean relative coordinate of K^* mesons:

$$\left\langle \sqrt{\mathbf{r}^2} \right\rangle_{K^*} = 0.79\ fm. \tag{73}$$

With the help of this relation we can estimate the mean charge radius of K^* mesons

$$\sqrt{\langle r_{K^*}^2 \rangle} \simeq \frac{\mu_2}{\mu_1 + \mu_2} \left\langle \sqrt{\mathbf{r}^2} \right\rangle_{K^*} = 0.54 \left\langle \sqrt{\mathbf{r}^2} \right\rangle_{K^*} = 0.43\ fm. \tag{74}$$

The experimental data of the mean charge radius of $K^\pm$ mesons having the analogous quark structure as K^* mesons are

$$\sqrt{\langle r_{K^\pm}^2 \rangle} = (0.53 \pm 0.05)\ fm \quad [46],$$

[2] The relationship $\sqrt{\langle \mathbf{r}^2 \rangle} \simeq \left\langle \sqrt{\mathbf{r}^2} \right\rangle$ is confirmed by the numerical calculations

$$\left\langle r^2_{K^\pm} \right\rangle = (0.34 \pm 0.05)\ fm^2 \quad [45],$$

and for neutral K^0 mesons [46]

$$\sqrt{\langle r^2_{K^0} \rangle} = (0.28 \pm 0.09)\ fm.$$

Expression (69) gives a value for the charge radius of K^*-mesons, that is close to experimental value for $K^\pm$-mesons; experimental data for the charge radius of K^*-mesons is not available.

The first perturbative one-gluon exchange contribution to the Hamiltonian determines the spin-spin correction such as the Breit-Fermi hyperfine interaction [19]. The spin-spin interaction is important to explain the Nambu-Goldstone phenomenon which takes place for $l = s = 0$ channel. To take into account spin-spin and spin-orbit interactions one needs to use the general expression (23) and calculate averaged Wilson integral with insertions [17].

Using the Hamiltonian (44) we estimated in [31, 36] the electromagnetic polarizabilities of mesons on the basis of the expansion in small electromagnetic fields **E**, **H**. The same procedure will be used bellow for nucleons.

5 Green's Function of Three-Quark System

Now we consider baryons as a three-quark system. Let us consider the Lorentz-covariant and gauge invariant combination of a three quark, colorless system (baryon) [17]

$$X_B(x, y, z, C_i) = \epsilon_{abc} \left[\Phi(Z_0, x) q(x)\right]_a \left[\Phi(Z_0, y) q(y)\right]_b \left[\Phi(Z_0, z) q(z)\right]_c , \quad (75)$$

where $q(x)$ is a quark bispinor; a, b, c are colour indexes so that

$$\left[\Phi(Z_0, x) q(x)\right]_a = \Phi_{aa'}(Z_0, x) q_{a'}(x)$$

and ϵ_{abc} is the Levi-Civita symbol ($\epsilon_{123} = 1$). The parallel transporter $\Phi(Z_0, x)$ is given by Eq. (2). The points x, y, z are the initial coordinates of three quarks with fields $q(x)$, $q(y)$ and $q(z)$, and the point Z_0 is arbitrary. Later, the position of Z_0 will be defined by requiring to have the minimal area for the world surface of three-quark system [17].

The two particle quantum Green function of a baryon is given by [17]:

$$G(xyz, x'y'z') = \langle X_B(x, y, z, C_i) \bar{X}_B(x', y', z', C'_i) \rangle, \quad (76)$$

were x', y',z' are final positions of three quarks; $\bar{X}_B(x', y', z', C'_i)$ corresponds to the final state of a baryon:

$$\bar{X}_B(x', y', z', C'_i) = \epsilon_{mnk} \left[\bar{q}(x')\Phi(x', Z'_0)\right]_m \left[\bar{q}(y')\Phi(y', Z'_0)\right]_n \left[\bar{q}(z')\Phi(z', Z'_0)\right]_k , \tag{77}$$

where $\bar{q} = q^+\gamma_4$. Using the generating functional (6) the Green function (76) takes the form

$$\begin{gathered} G(xyz, x'y'z') = \int DA_\mu \Big[\epsilon_{abc}\epsilon_{mnk}\Phi_{aa'}(Z_0, x)\Phi_{bb'}(Z_0, y)\Phi_{cc'}(Z_0, z) \\ \times\Phi_{m'm}(x', Z'_0)\Phi_{n'n}(y', Z'_0)\Phi_{k'k}(z', Z'_0) \\ \times\delta^6/\delta\bar{\eta}_{a'}(x)\delta\bar{\eta}_{b'}(y)\delta\bar{\eta}_{c'}(z)\delta\eta_{m'}(x')\delta\eta_{n'}(y')\delta\eta_{k'}(z')Z[\bar{\eta}, \eta]\Big]_{\eta=\bar{\eta}=0} . \end{gathered} \tag{78}$$

In Eq. (78) Z_0 and Z'_0 are the initial and final positions of the string junction, respectively. The total surface which consists of world motions of quarks and the path of the string junction must be minimal. This requirement defines the path of the string junction [17]. Because the path-integral in Eq. (78) is a Gaussian, it is integrated over quark fields $\bar{q}$, q .

Taking into account Eq. (9) and calculating the variation derivatives in Eq. (78) we find the quantum Green function of a baryon:

$$\begin{gathered} G(xyz, x'y'z') = \int DA_\mu \det(-\gamma_\mu D_\mu - m) \exp\{iS(A)\}\, \epsilon_{abc}\epsilon_{mnk} \\ \times\Big[S^\Phi_{bm}(y, x')S^\Phi_{an}(x, y')S^\Phi_{ck}(z, z') - S^\Phi_{am}(x, x')S^\Phi_{bn}(y, y')S^\Phi_{ck}(z, z') \\ +S^\Phi_{cm}(z, x')S^\Phi_{bn}(y, y')S^\Phi_{ak}(x, z') - S^\Phi_{cm}(z, x')S^\Phi_{an}(x, y')S^\Phi_{bk}(y, z') \\ +S^\Phi_{am}(x, x')S^\Phi_{cn}(z, y')S^\Phi_{bk}(y, z') - S^\Phi_{bm}(y, x')S^\Phi_{cn}(z, y')S^\Phi_{ak}(x, z')\Big], \end{gathered} \tag{79}$$

where we introduce the following notation for the covariant Green function

$$S^\Phi_{am}(x, x') = \Phi_{aa'}(Z_0, x)S_{a'm'}(x, x')\Phi_{m'm}(x', Z'_0). \tag{80}$$

As in the case of mesons the functional determinant in Eq. (79) gives the contribution of the additional quark loops. The presence of different terms in Eq. (80) is connected with the permutations of quark fields because the quantum Green function being considered. As different terms in Eq. (79) have the same structure, we consider in detail only one term. Neglecting the functional determinant we find the approximate expression for the baryon Green function

$$G_1(xyz, x'y'z') = -\int DA_\mu \exp\{iS(A)\}\, \epsilon_{abc}\epsilon_{mnk} S^{\Phi}_{am}(x,x') S^{\Phi}_{bn}(y,y') S^{\Phi}_{ck}(z,z'). \tag{81}$$

Expression (81) is the basic formula for deriving effective action for baryons.

6 Effective Action for Baryons

We consider baryons in external electromagnetic fields. Inserting Eq. (21) into Eq. (80) we find

$$S^{\Phi}_{am}(x,x') = -i\int_0^\infty ds \int_{z(0)=x'}^{z(s)=x} Dz\left(m_1 - \frac{i}{2}\gamma_\mu \dot{z}_\mu(t)\right) P_\Sigma$$
$$\times \exp\Big\{ i\int_0^s dt\Big[\frac{1}{4}\dot{z}^2_\mu(t) - m_1^2 + e_1\dot{z}_\mu(t) A^{el}_\mu(z)$$
$$+\Sigma_{\mu\nu}(e_1 F^{el}_{\mu\nu} + gF_{\mu\nu})\Big]\Big\}\,(\Phi_{C_x}(Z_0, Z'_0))_{am}\,, \tag{82}$$

where the contour C_x in Eq. (82) consists of lines between Z_0, x and Z'_0, x' and path $z_\mu(t)$. Using the expression (82) for each quark, the baryon Green function (81) becomes

$$G_1(xyz, x'y'z') = i\int_0^\infty \prod_j ds_j \int \prod_j Dz^{(j)}\left(m_j - \frac{i}{2}\gamma_\mu \dot{z}^{(j)}_\mu(t_j)\right)$$
$$\times P_\Sigma \prod_j \exp\left\{\int_0^{s_j} dt_j \Sigma_{\mu\nu}\frac{\delta}{\delta\sigma^{(j)}_{\mu\nu}(t_j)}\right\}$$
$$\times \exp\Big\{ i\sum_j \int_0^{s_j} dt_j\Big[\frac{1}{4}\left(\dot{z}^{(j)}_\mu(t_j)\right)^2 - m_j^2 + e_j\dot{z}^{(j)}_\mu(t_j) A^{el}_\mu(z^{(j)})$$
$$+e_j\Sigma_{\mu\nu}F^{el}_{\mu\nu}\Big]\Big\}\langle W(C_xC_yC_z)\rangle_A, \tag{83}$$

where $j = 1,2,3$; e_j is the charge of the j-th quark; the boundary conditions $z^{(1)}_\mu(0) = x'_\mu$, $z^{(1)}_\mu(s_1) = x_\mu$, $z^{(2)}_\mu(0) = y'_\mu$, $z^{(2)}_\mu(s_2) = y_\mu$, $z^{(3)}_\mu(0) = z'_\mu$, $z^{(3)}_\mu(s_3) = z_\mu$ are used here and the Wilson loop is given by (see [17])

$$\langle W(C_xC_yC_z)\rangle_A = \epsilon_{abc}\epsilon_{mnk}\langle\Big(\Phi_{C_x}(Z_0, Z'_0)\Big)_{am}$$

$$\times \left(\Phi_{C_y}(Z_0, Z_0')\right)_{bn} (\Phi_{C_z}(Z_0, Z_0'))_{ck}\rangle_A. \tag{84}$$

We took into account relations such as (28) (see [17]). Contours C_x, C_y, C_z correspond to three quarks which have paths $z_\mu^{(1)}$, $z_\mu^{(2)}$, $z_\mu^{(3)}$ and masses m_1, m_2, m_3, respectively. Relationship (83) is the generalization of one [17] for the case of quarks placed in external electromagnetic fields which possess spins.

We imply further that the average distance between quarks $\langle r\rangle$ is greater than the time fluctuations (in units $c = \hbar = 1$) of the gluonic fields T_g: $\langle r\rangle > T_g$. This condition is valid not only for asymptotic baryon states of the Regge trajectories with large angular momenta of the baryon but also for lower baryon states. The asymptotic of the average Wilson loop integral obeys then the area law and is given by (in the Minkowski space):

$$\langle W(C_x C_y C_z)\rangle_A = \exp\left\{-i\sigma\left(S_1 + S_2 + S_3\right)\right\}, \tag{85}$$

where σ is the string tension and S_j $(j = 1, 2, 3)$ is the minimal surface bounded by the trajectories of the quarks q_j and string junction Z_0. The path of the string junction is defined by the requirement that the sum $S_1 + S_2 + S_3$ is minimal [17]. Following [17], new variables are introduced:

$$\tau = \frac{t_1 T}{s_1} = \frac{t_2 T}{s_2} = \frac{t_3 T}{s_3}, \qquad \mu_j = \frac{T}{2s_j}, \tag{86}$$

where τ means the proper time for every quark and μ_j $(j = 1, 2, 3\ )$ is the dynamical mass of the $j-$th quark. Using Eqs. (85), (86) from Eq. (83) we arrive at

$$G_1(xyz, x'y'z') = -i\frac{T^3}{8}\int_0^\infty \prod_j \frac{d\mu_j}{\mu_j^2}\int\prod_j Dz^{(j)}\left(m_j - i\mu_j\gamma_\mu \dot{z}_\mu^{(j)}(\tau)\right)$$

$$\times P_\Sigma \prod_j \exp\left\{\frac{1}{2\mu_j}\Sigma_{\mu\nu}\int_0^T d\tau \frac{\delta}{\delta\sigma_{\mu\nu}^{(j)}(\tau)}\right\}$$

$$\times \exp\Bigg\{i\int_0^T d\tau \sum_j\Bigg[\frac{1}{2}\mu_j\left(\dot{z}_\mu^{(j)}(\tau)\right)^2 - \frac{m_j^2}{2\mu_j} + e_j \dot{z}_\mu^{(j)}(\tau) A_\mu^{el}(z^{(j)})$$

$$+ \frac{e_j}{2\mu_j}\Sigma_{\mu\nu}F_{\mu\nu}^{el}\Bigg] - i\sigma\left(S_1 + S_2 + S_3\right)\Bigg\}. \tag{87}$$

The integral in the last exponential factor of Eq. (87) represents the effective action for three quark system (baryon) taking into account the spins of quarks. So T is the time of the observation, τ is the proper time of quarks; m_j and μ_j are the current and dynamical masses of $j-$th quark. It follows from Eq. (87) that there is integration over dynamical masses μ_j to get the Green function of the three quark system. Pre-exponential factors in Eq. (87) allow us to calculate in principle the spin-spin and spin-orbital contributions to the effective action. As a first approximation (see [17]) we neglect the short-range spin corrections and consider therefore scalar quarks. The terms $e_j\Sigma_{\mu\nu}F^{el}_{\mu\nu}$ which describe the interaction of the magnetic field with the spins of quarks will be omitted. With this assumption we arrive at the effective action for baryons in external electromagnetic fields

$$B = \int_0^T d\tau \sum_j \Bigg[\frac{1}{2}\mu_j \left(\dot{z}^{(j)}_\mu(\tau)\right)^2 - \frac{m_j^2}{2\mu_j}$$

$$+e_j \dot{z}^{(j)}_\mu(\tau) A^{el}_\mu(z^{(j)})\Bigg] - \sigma\left(S_1+S_2+S_3\right). \tag{88}$$

The case when electromagnetic fields are absent was considered in [17]. The terms which describe the interaction of quarks with electromagnetic fields are essential for us because we are going to calculate electromagnetic characteristics of nucleons. It is convenient to introduce new variables R_μ, ξ_μ and η_μ instead of $z^{(j)}_\mu$ in accordance with relationships [17]:

$$z^{(1)}_\mu = R_\mu + \left(\frac{\mu\mu_3}{M\left(\mu_1+\mu_2\right)}\right)^{1/2}\xi_\mu - \left(\frac{\mu\mu_2}{\mu_1\left(\mu_1+\mu_2\right)}\right)^{1/2}\eta_\mu,$$

$$z^{(2)}_\mu = R_\mu + \left(\frac{\mu\mu_3}{M\left(\mu_1+\mu_2\right)}\right)^{1/2}\xi_\mu + \left(\frac{\mu\mu_1}{\mu_2\left(\mu_1+\mu_2\right)}\right)^{1/2}\eta_\mu,$$

$$z^{(3)}_\mu = R_\mu - \left(\frac{\mu\left(\mu_1+\mu_2\right)}{M\mu_3}\right)^{1/2}\xi_\mu, \tag{89}$$

where R_μ is the center of mass coordinate of a baryon; ξ_μ and η_μ are relative coordinates of quarks, $M = \mu_1+\mu_2+\mu_3$ is the sum of dynamical masses of quarks. The arbitrary mass parameter μ in Eq. (89) defines the scale of relative coordinates ξ_μ and η_μ. After the substitution (89), the measure $\prod_j Dz^{(j)}$ transforms into $DRD\eta D\xi$ in path-integral (87).

Let us consider the uniform and constant external electromagnetic fields. Then the vector-potential of electromagnetic fields can be represented as

$$A^{el}_\nu(z^{(j)}) = \frac{1}{2}F^{el}_{\mu\nu}z^{(j)}_\mu.$$

Inserting Eqs. (89) into Eq. (88) we find the effective action for the three quark system (see [37]) in the form

$$B=\int_0^T d\tau\left[\frac{M}{2}\dot{R}_\nu^2+\frac{\mu}{2}\left(\dot{\xi}_\nu^2+\dot{\eta}_\nu^2\right)-\sum_j\frac{m_j^2}{2\mu_j}\right]-\sigma\left(S_1+S_2+S_3\right)+\Delta B, \quad (90)$$

$$\Delta B=\frac{1}{2}F_{\nu\mu}^{el}\int_0^T d\tau\Big[e\dot{R}_\mu R_\nu+\lambda\dot{\xi}_\mu\xi_\nu+\rho\dot{\eta}_\mu\eta_\nu+\gamma\left(\dot{R}_\mu\xi_\nu+\dot{\xi}_\mu R_\nu\right)$$

$$+\delta\left(\dot{R}_\mu\eta_\nu+\dot{\eta}_\mu R_\nu\right)+\delta\sqrt{\frac{\mu\mu_3}{(\mu_1+\mu_2)\,M}}\left(\dot{\xi}_\mu\eta_\nu+\dot{\eta}_\mu\xi_\nu\right)\Big], \quad (91)$$

where we introduce parameters:

$$\gamma=\sqrt{\frac{\mu}{M}}\left[(e_1+e_2)\sqrt{\frac{\mu_3}{\mu_1+\mu_2}}-e_3\sqrt{\frac{\mu_1+\mu_2}{\mu_3}}\right],$$

$$\delta=\sqrt{\frac{\mu}{\mu_1+\mu_2}}\left(e_2\sqrt{\frac{\mu_1}{\mu_2}}-e_1\sqrt{\frac{\mu_2}{\mu_1}}\right),$$

$$\rho=\frac{\mu}{\mu_1+\mu_2}\left(\frac{e_1\mu_2}{\mu_1}+\frac{e_2\mu_1}{\mu_2}\right),$$

$$\lambda=\frac{\mu}{M}\left[\frac{(e_1+e_2)\,\mu_3}{\mu_1+\mu_2}+\frac{e_3\,(\mu_1+\mu_2)}{\mu_3}\right], \quad (92)$$

and $e=e_1+e_2+e_3$ is the charge of a baryon. As a particular case, when electromagnetic fields are absent ($\Delta B=0$) we arrive at the action derived in [17]. It follows from Eq. (90) that the center of mass coordinate R_μ is separated from relative coordinates ξ_μ and η_μ and μ plays the role of the mass of the ξ_μ, η_μ excitations. Following [17], the straight line approximation for strings and the asymmetric quark-diquark structure of baryons will be assumed. The asymmetric configuration (see also [35,38]) means that two quarks $q^{(1)}$ and $q^{(2)}$ are near each other and quark $q^{(3)}$ is farther from them. This case is preferable [35,38] and the slope of linear baryon Regge trajectories is the same as for mesons [17]. Then $\sqrt{\xi^2}\gg\sqrt{\eta^2}$ ($\xi^2=\xi_1^2+\xi_2^2+\xi_3^2$) and the coordinate η_μ can be ignored. We neglect therefore the surfaces S_1, S_2 and assume for S_3 the following expression [17]:

$$S_3=b\int_0^T d\tau\sqrt{\xi^2}, \quad (93)$$

where

$$b=\sqrt{\frac{\mu\left(\mu_1+\mu_2\right)}{M\mu_3}}+\sqrt{\frac{\mu\mu_3}{M\left(\mu_1+\mu_2\right)}}. \tag{94}$$

Equation (93) takes into account the confinement of quarks and gives the linear potential between quarks. Using the definition $B=\int_0^T d\tau\mathcal{L}$, where $\mathcal{L}$ is the Lagrangian, from Eq. (90) we arrive at the effective Lagrangian for baryons

$$\mathcal{L}_{eff}=\frac{M}{2}\dot{R}_\nu^2+\frac{\mu}{2}\dot{\xi}_\nu^2-\sum_j\frac{m_j^2}{2\mu_j}-\sigma b\sqrt{\xi^2}+\mathcal{L}^{el}. \tag{95}$$

Here we neglect the coordinate η_μ and introduce the notation:

$$\mathcal{L}^{el}=\frac{1}{2}F_{\nu\mu}^{el}\left[e\dot{R}_\mu R_\nu+\lambda\dot{\xi}_\mu\xi_\nu+\gamma\left(\dot{R}_\mu\xi_\nu+\dot{\xi}_\mu R_\nu\right)\right], \tag{96}$$

Lagrangian $\mathcal{L}^{el}$ describes the electromagnetic interaction of the string. It follows from Eq. (89) that

$$z_\mu^{(3)}-\frac{1}{2}\left(z_\mu^{(1)}+z_\mu^{(2)}\right)=-b\xi_\mu+\frac{1}{2}\sqrt{\frac{\mu}{\mu_1+\mu_2}}\left(\frac{\mu_2-\mu_1}{\sqrt{\mu_1\mu_2}}\right)\eta_\mu. \tag{97}$$

As the second term in Eq. (97) is small, the coordinate ξ_μ is proportional to the "distance" between quark $q^{(3)}$ and the center of mass of quarks $q^{(1)}$ and $q^{(2)}$ which form a diquark. At the large time T limit $\xi_4=0$, $R_4=i\tau$ [17] and Lagrangian (96) takes the form

$$\mathcal{L}^{el}=e\left(\mathbf{R}\mathbf{E}\right)+\gamma\left(\xi\mathbf{E}\right)-\frac{1}{2}\epsilon_{mnk}H_k\left(e\dot{R}_mR_n+\lambda\dot{\xi}_m\xi_n+\gamma(\dot{R_m\xi_n}+\dot{\xi}_mR_n)\right), \tag{98}$$

where the electric field $E_k=iF_{k4}$ and magnetic field $H_k=(1/2)\epsilon_{kmn}F_{mn}$. To clarify the physical meaning of the terms in Eq. (98), let us consider the dipole moment of quarks. Using the definition of the electric dipole moment and Eq. (89) we have

$$\mathbf{d}=\sum_j e_j\mathbf{z}^{(j)}=e\mathbf{R}+\gamma\xi+\delta\eta. \tag{99}$$

So, first two terms in Eq. (98) (neglecting the coordinate η_μ) describe the interaction of the dipole moment of quarks with the electric field in accordance with the expression for the potential energy: $U = -(\mathbf{dE})$. The magnetic moment is given by

$$m_k = \frac{1}{2}\epsilon_{mnk}\sum_j e_j z_m^{(j)}\dot{z}_n^{(j)} = \frac{1}{2}\epsilon_{mnk}\Big[eR_m\dot{R}_n + \lambda\xi_m\dot{\xi}_n + \gamma\left(R_m\dot{\xi}_n + \xi_m\dot{R}_n\right)$$

$$+\rho\eta_m\dot{\eta}_n + \delta\left(R_m\dot{\eta}_n + \eta_m\dot{R}_n\right) + \delta\sqrt{\frac{\mu\mu_3}{(\mu_1+\mu_2)\,M}}\left(\xi_m\dot{\eta}_n + \eta_m\dot{\xi}_n\right)\Big]. \quad (100)$$

It follows from Eqs. (91), (98) that there is an interaction of the magnetic field with the magnetic moment in such a way that the interaction energy is $U = -(\mathbf{mH})$. So Lagrangian (98) describes the interaction of electric and magnetic moments of baryons with electric and magnetic fields, respectively.

7 Mean Relative Coordinates of quarks in Nucleons

Now we estimate the mean size of baryons. At the large time T limit $\dot{\xi}_0 = 0$ ($\xi_4 = i\xi_0$), $R_4 = i\tau$ [17] and therefore only three dimensional quantities R_k and ξ_k are important. From Eq. (98) we find three momenta corresponding to the center of mass coordinate R_k and relative coordinate ξ_k :

$$\Pi_k = \frac{\partial\mathcal{L}_{eff}}{\partial\dot{R}_k} = M\dot{R}_k - \frac{1}{2}\epsilon_{knm}H_m\left(eR_n + 2\gamma\xi_n\right),$$

$$\pi_k = \frac{\partial\mathcal{L}_{eff}}{\partial\dot{\xi}_k} = \mu\dot{\xi}_k - \frac{1}{2}\epsilon_{knm}H_m\left(\lambda\xi_n + 2\gamma R_n\right). \quad (101)$$

Here we take into account that Lagrangian is defined within the accuracy of the total derivative on time. Then the effective Hamiltonian $\mathcal{H}_{eff} = \Pi_k\dot{R}_k + \pi_k\dot{\xi}_k - \mathcal{L}_{eff}$ corresponding to the quark-diquark structure of a baryon is given by

$$\mathcal{H}_{eff} = \sum_j \frac{m_j^2}{2\mu_j} + \frac{M}{2} + \frac{M}{2}\dot{R}_k^2 + \frac{\mu}{2}\dot{\xi}_k^2 + \sigma b\sqrt{\xi^2} - e\left(\mathbf{ER}\right) - \gamma\left(\mathbf{E}\xi\right), \quad (102)$$

The Hamiltonian for baryons Eq. (102) looks like the one for mesons (44) because we consider basically the string between quark and diquark. Therefore the calculations of mean coordinates and electromagnetic polarizabilities is the same [31,36]. But in the case of baryons there are more parameters and the analysis is more complicated.

With the help of Eq. (101) the effective Hamiltonian (102) is rewritten as

$$\mathcal{H}_{eff} = \sum_j \frac{m_j^2}{2\mu_j} + \frac{M}{2} + \frac{1}{2M}\left[\mathbf{\Pi} + \frac{e}{2}\left(\mathbf{R}\times\mathbf{H}\right) + \gamma\left(\xi\times\mathbf{H}\right)\right]^2$$

$$+\frac{1}{2\mu}\left[\pi + \frac{\lambda}{2}\left(\xi\times\mathbf{H}\right) + \gamma\left(\mathbf{R}\times\mathbf{H}\right)\right]^2 + \sigma b\sqrt{\xi^2} - e\left(\mathbf{E}\mathbf{R}\right) - \gamma\left(\mathbf{E}\xi\right), \quad (103)$$

where $\left(\xi\times\mathbf{H}\right)_k = \epsilon_{mnk}\xi_m H_n$. The mass of a baryon $\mathcal{M}(\mu_j)$ is defined here as a solution to equation

$$\mathcal{H}_{eff}\Phi = \mathcal{M}(\mu_j)\Phi. \quad (104)$$

In according to the Noether theorem, the momentum $\mathbf{\Pi}$ is conserved and we can put $\mathbf{R} = \mathbf{\Pi} = 0$ in Eq. (103). To find the solution to Eq. (104) we can use the substitution $\pi_k = -i\partial/\partial\xi_k$. Then the mass of a baryon $\mathcal{M}(\mu_j)$ is given by

$$\mathcal{M}(\mu_j) = \sum_j \frac{m_j^2}{2\mu_j} + \frac{M}{2} + \epsilon(\mu_j, \mathbf{E}, \mathbf{H}), \quad (105)$$

where $\epsilon(\mu_j, \mathbf{E}, \mathbf{H})$ is the eigenvalue of the equation

$$\left\{\frac{1}{2\mu}\left[-i\frac{\partial}{\partial\xi} - \frac{\lambda}{2}\left(\xi\times\mathbf{H}\right)\right]^2 + \frac{\gamma^2}{2M}\left(\xi\times\mathbf{H}\right)^2 + \sigma b\sqrt{\xi^2} - \gamma\left(\mathbf{E}\xi\right)\right\}\Phi$$

$$= \epsilon(\mu_j, \mathbf{E}, \mathbf{H})\Phi. \quad (106)$$

The term $\left(\gamma^2/(2M)\right)\left(\xi\times\mathbf{H}\right)^2$ in Eq. (106) is due to the recoil of the string. In nonrelativistic models the effect of the recoil was studied in [39] (see also [22]). As we neglected the spin of baryons here, there is no interaction of spin with the external magnetic field. It is not difficult to take into account such interaction. Eq. (106) is like the equation for mesons [36].

If $\mathbf{E} = \mathbf{H} = 0$, we arrive from Eq. (106) at

$$\left(-\frac{1}{2\mu}\frac{\partial^2}{\partial\xi_i^2} + \sigma b\sqrt{\xi^2}\right)\Phi = \epsilon(\mu_j)\Phi. \quad (107)$$

Equation (107) has the same structure as Eq. (51). The eigenvalue of Eq. (107) is

$$\epsilon(\mu_j) = (2\mu)^{-1/3}\,(b\sigma)^{2/3}\,a(n) = (\sigma)^{2/3}a(n)\left[\frac{M}{2\mu_3\,(\mu_1+\mu_2)}\right]^{1/3}. \qquad (108)$$

The condition of the minimum of the baryon mass (105) ($\partial\mathcal{M}(\mu_j)/\mu_j = 0$) at $m_1 = m_2$ (and $\mu_1 = \mu_2$), with the help of Eq. (105) gives the dynamical mass of a diquark:

$$\mu_3^{(0)} = \mu_1^{(0)} + \mu_2^{(0)} = \sqrt{\sigma}\left[\frac{a(n)}{3}\right]^{3/4}. \qquad (109)$$

This value is different from one [17] obtained for large angular momentum. Using Eq. (109) we arrive from Eq. (105) at the expression for the mass of a baryon (see also [17]):

$$\mathcal{M}(\mu_j) = \frac{m_3^2 + 4m_1^2}{2\mu_3^{(0)}} + 4\mu_3^{(0)}. \qquad (110)$$

To estimate the baryon mass, the value of the string tension $\sigma = 0.15$ GeV^2 will be used [17]. Neglecting the small current masses of quarks m_j we find from Eqs. (109), (110) the mass of a diquark for $n = 1$: $\mu_3^{(0)} = 320$ MeV and the nucleon mass : $\mathcal{M}(\mu_j) = 1.28$ GeV [37]. This value of the nucleon mass is a little greater then real nucleon mass because spin-spin and spin-orbit forces were omitted.

Now we consider the mean relative coordinates of nucleons on the basis of the virial theorem which gives (as for the case of mesons [36]) the mean potential energy $\langle U\rangle = 2\langle T\rangle$, where $\langle T\rangle$ is the mean kinetic energy. Then using the relation $\langle T\rangle + \langle U\rangle = \epsilon(\mu_j)$, we arrive at

$$\langle U\rangle = \frac{2}{3}\epsilon(\mu_j) = \frac{2}{3}(2\mu)^{-1/3}(b\sigma)^{2/3}a(n). \qquad (111)$$

Comparing Eq. (111) with the relation $\langle U\rangle = b\sigma\langle\sqrt{\xi^2}\rangle$ gives the following expression

$$\langle\sqrt{\xi^2}\rangle = \frac{2}{3}(2\mu b\sigma)^{-1/3}a(n) = \sigma^{-1/4}\sqrt{\frac{2}{\mu}}\left(\frac{a(n)}{3}\right)^{9/8}. \qquad (112)$$

In accordance with Eq. (97) the size of the nucleon is characterized by the value $\mid \mathbf{z}^{(3)} - (1/2)(\mathbf{z}^{(1)} + \mathbf{z}^{(2)}) \mid \simeq \mid b\xi \mid$. Introducing the notation $\mathbf{r}_b = b\xi$, from Eq. (112) we have

$$\langle \sqrt{\mathbf{r}_b^2} \rangle = \frac{2}{\sqrt{\sigma}} \left(\frac{a(n)}{3} \right)^{3/4}. \tag{113}$$

The same expression was found for mesons (see Eq. (70)). For the quark-diquark system the string tension coincides with those of mesons and therefore the quark-diquark system has approximately the same size as mesons. Using $\sigma = 0.15$ GeV^2 and $a(1) = 2.2281$ we find the mean size of the nucleons

$$\langle \sqrt{\mathbf{r}_b^2} \rangle = 0.84 \ fm. \tag{114}$$

The experimental value of the charge radii of the proton and neutron are

$$\sqrt{\langle r_p^2 \rangle} = 0.86 \ fm \quad [47],$$

$$\sqrt{\langle r_n^2 \rangle} = (-0.113 \pm 0.003) \ fm \quad [48].$$

In accordance with Eq. (89) the center of mass of the quark-diquark system is situated in the center between quark $q^{(3)}$ and diquark $\left(q^{(1)}, q^{(2)}\right)$ and the mean radius of a nucleon is $(1/2) \langle \sqrt{\mathbf{r}_b^2} \rangle = 0.42$ fm which is the reasonable value. It should be noted that charge radii of hadrons are defined from electromagnetic formfactors.

8 Electric Polarizabilities of Nucleons

To evaluate electric polarizabilities of nucleons we consider the case when $\mathbf{H} = 0$, $\mathbf{E} \neq 0$. It is possible to assume as an approximation that $\mathbf{E} \parallel \xi$ [37], i.e. external electric field is parallel to the string which connects quark $q^{(3)}$ with diquark $\left(q^{(1)}, q^{(2)}\right)$. So we neglect the rotation of the string. It is justified only for the ground state when the orbital quantum number $l = 0$. Introducing the effective string tension

$$\sigma_{eff} = \sigma - \frac{\gamma}{b} E, \tag{115}$$

we arrive from Eq. (108) at the eigenvalue

$$\epsilon(\mu_j, \mathbf{E}) = (2\mu)^{-1/3} \left(b\sigma_{eff}\right)^{2/3} a(n).$$

From (109), (110), by neglecting the small terms containing the current masses we find the mass of a baryon

$$\mathcal{M}(\mu_j, \mathbf{E}) = 4\sqrt{\sigma_{eff}} \left[\frac{a(n)}{3}\right]^{3/4}, \tag{116}$$

which depends on the external electric field. Inserting Eq. (115) into Eq. (116) and expanding it in a small electric field one yields

$$\mathcal{M}(\mu_j, \mathbf{E}) \simeq \left[\frac{a(n)}{3}\right]^{3/4} \left(4\sqrt{\sigma} - \frac{q}{\sqrt{\sigma}}E - \frac{q^2}{8}\sigma^{-3/2}E^2\right), \tag{117}$$

where Eq. (109) was used and $q = e_1 + e_2 - e_3$. We write here only terms of the expansion $\mathcal{M}(\mu_j, \mathbf{E})$ in small electric field up to E^2. The first term in Eq. (117) gives the mass of a baryon. The second one is connected with the potential energy of a dipole moment of quarks in the external electric field $U = -\mathbf{dE}$. From Eq. (99) when the center of mass coordinate $\mathbf{R} = 0$ and $\mid \xi \mid \gg \mid \eta \mid$, the dipole moment of quark-diquark system is $\mathbf{d} \simeq \gamma\xi = (\gamma/b)\mathbf{r}_b$. Comparing the potential energy of a dipole $U = -(\gamma/b)r_b E$ (at $\mathbf{E} \parallel \mathbf{r}$) with the second term of Eq. (117): $-(q/\sqrt{\sigma})[a(n)/3]^{3/4}E$ we arrive at the expression for the mean relative coordinate $r_b = (2/\sqrt{\sigma})[a(n)/3]^{3/4}$ which coincides with Eq. (113). Here we define the electric dipole moment of quark-diquark system more precisely as compared with the letter [37] and as a result the mean relative coordinate of a baryon Eq. (113) coincides with that for a meson. The third term in Eq. (117) describes the potential energy due to the electric polarizability of a baryon (see Eq. (155) in Appendix).

From Eq. (117) by comparing the quadratic term in $\mathbf{E}$ with Eq. (155) we arrive at the electric polarizability of a baryon:

$$\alpha = \frac{q^2}{4}\left[\frac{a(n)}{3}\right]^{3/4} \sigma^{-3/2}. \tag{118}$$

Let us consider the estimation of the electric polarizability for the proton $p = uud$. There are two possibilities for a proton as a quark-diquark system: a) the quark $q^{(3)} = d$ and diquark $(q^{(1)}q^{(2)}) = (uu)$, so the electric charges $e_1 = e_2 = (2e)/3$, $e_3 = -e/3$ and parameter $q = e_1 + e_2 - e_3 = (5e)/3$; b) the quark $q^{(3)} = u$, diquark $(q^{(1)}q^{(2)}) = (ud)$ and the electric charges $e_1 = e_3 = (2e)/3$, $e_2 = -e/3$ and parameter $q = e_1 + e_2 - e_3 = -e/3$. It should be noted that there are no permutations of quarks here which occur in the Green function Eq. (79) due to the Pauli principle. Using the value of the string tension $\sigma = 0.15$ GeV^2 and $a(1) = 2.3381$ from Eq. (118) we find the static polarizability of a proton in Gaussian units for two cases

$$a)\ \alpha_p = 5.56 \times 10^{-4}\ fm^3, \qquad b)\ \alpha_p = 0.22 \times 10^{-4}\ fm^3. \tag{119}$$

From Eq. (161) (see Appendix) $\Delta\alpha_p = (4.5 \pm 0.1) \times 10^{-4}\ fm^3$, and the total electric polarizability of a proton becomes (see Eq. (158))

$$a)\ \bar{\alpha}_p = 10 \times 10^{-4}\ fm^3, \qquad b)\ \bar{\alpha}_p = 4.7 \times 10^{-4}\ fm^3. \tag{120}$$

The configuration of a proton a), when the quark $q^{(3)} = d$ and diquark $(q^{(1)}q^{(2)}) = (uu)$ is more favorable as the experimental values of electric polarizability are

$$\bar{\alpha}_p^{\exp} = (10.9 \pm 2.2 \pm 1.3) \times 10^{-4}\ fm^3 \quad [49],$$

$$\bar{\alpha}_p^{\exp} = (10.6 \pm 1.2 \pm 1.0) \times 10^{-4}\ fm^3 \quad [50],$$

$$\bar{\alpha}_p^{\exp} = (9.8 \pm 0.4 \pm 1.1) \times 10^{-4}\ fm^3 \quad [51].$$

So in the case a) we have a good agreement with experimental data.

For the neutron $n = udd$ there are also two possibilities: a) the quark $q^{(3)} = u$ and diquark $(q^{(1)}q^{(2)}) = (dd)$, so the electric charges $e_1 = e_2 = -e/3$, $e_3 = (2e)/3$ and parameter $q = e_1 + e_2 - e_3 = -(4e)/3$; b) the quark $q^{(3)} = d$, diquark $(q^{(1)}q^{(2)}) = (ud)$ and the electric charges $e_3 = e_2 = -e/3$, $e_1 = (2e)/3$ and parameter $q = e_1 + e_2 - e_3 = (2e)/3$. Inserting these parameters into Eq. (118) one gives

$$a)\ \alpha_n = 3.56 \times 10^{-4}\ fm^3, \qquad b)\ \alpha_n = 0.89 \times 10^{-4}\ fm^3. \tag{121}$$

For the neutron, in accordance with Eq. (161) $\Delta\alpha_n = 0.62 \times 10^{-4}$ fm^3 and the generalized electric polarizability of a neutron is given by

$$a)\ \bar{\alpha}_n = 4.2 \times 10^{-4}\ fm^3, \qquad b)\ \bar{\alpha}_n = 1.5 \times 10^{-4}\ fm^3. \tag{122}$$

The experimental situation for a neutron is more complicated as there are different experimental data:

$$\bar{\alpha}_n^{\exp} = (0.0 \pm 5) \times 10^{-4}\ fm^3 \quad [48], \tag{123}$$

$$\bar{\alpha}_n^{\exp} = (12.6 \pm 1.5 \pm 2.0) \times 10^{-4}\ fm^3 \quad [52]. \tag{124}$$

The recent experimental data (123) are close to both cases but the case b) with the quark $q^{(3)} = d$ and diquark $(q^{(1)}q^{(2)}) = (ud)$ is more favorable. Other experimental data [52] are close to the case a) in Eq. (122) with quark $q^{(3)} = u$ and diquark $(q^{(1)}q^{(2)}) = (dd)$ but the magnitude (122) a) is small. This experimental discrepancy does not allow us to choose reliably one of the possibilities: a) or b). The value of $\bar{\alpha}_n$ in the situation a) in Eq. (122) is close to one obtained in the oscillator nonrelativistic quark model [53-55,22]. The results of calculations in the framework of the dispersion sum rule and CHPT Eq. (167) are closer to the case a).

9 Diamagnetic Polarizabilities of Nucleons

In according to Eq. (103) for calculating the magnetic polarizability of nucleons one needs to compare it with Eq. (155). The effective Hamiltonian (103) (at $\mathbf{E} = 0$, $\mathbf{R} = \mathbf{\Pi} = 0$) can be cast into

$$\mathcal{H}_{eff} = \mathcal{H}_0 + \mathcal{H}_{int},$$

$$\mathcal{H}_0 = \sum_j \frac{m_j^2}{2\mu_j} + \frac{M}{2} - \frac{1}{2\mu}\frac{\partial^2}{\partial \xi_j^2} + \sigma b\sqrt{\xi^2},$$

$$\mathcal{H}_{int} = -\frac{\lambda}{2\mu}\mathbf{HL} + \left(\frac{\lambda^2}{8\mu} + \frac{\gamma^2}{2M}\right)\left[(\xi \times \mathbf{H})\right]^2, \tag{125}$$

where $L_k = -i\epsilon_{kmn}\xi_m\partial_n$ is the angular momentum and $\partial_n = \partial/\partial\xi_n$. Without loss of generality we can choose the direction of the magnetic field on the third axis, i.e. $\mathbf{H} = (0, 0, H)$. Then the Hamiltonian of an interaction of quarks with the magnetic field is given by

$$\mathcal{H}_{int} = -\frac{\lambda H}{2\mu}L_3 + H^2\left(\frac{\lambda^2}{8\mu} + \frac{\gamma^2}{2M}\right)\left(\xi_1^2 + \xi_2^2\right), \tag{126}$$

where $L_3 = i\left(\xi_2\partial_1 - \xi_1\partial_2\right)$. Considering the small external magnetic field, the perturbative theory can be applied. Using the perturbative method [56], within the accuracy of the second order, one arrives at the shift of the energy

$$\Delta\mathcal{E}_n = \langle n \mid \left[-\frac{\lambda H}{2\mu}L_3 + \left(\frac{\lambda^2}{8\mu} + \frac{\gamma^2}{2M}\right)H^2\xi^2 \sin\vartheta\right] \mid n\rangle$$

$$+\sum_{n'}\frac{|\langle n' \mid -(\lambda H L_3)/(2\mu) \mid n\rangle|^2}{\mathcal{E}_n-\mathcal{E}_{n'}}, \tag{127}$$

where ϑ is the angle between coordinate ξ and magnetic field $\mathbf{H}$. If the first term in Eq. (127) is not equal to zero then the second and third terms are smaller, and the main contribution to the energy comes from the first term. This occurs when the orbital quantum number $l > 0$. In the case $l = 0$, the shift of the energy due to the interaction with the magnetic field is defined by the second term in Eq. (127).

After averaging in Eq. (127) and taking into account the equation

$$\frac{1}{4\pi}\int \sin^2\vartheta d\Omega = \frac{2}{3}$$

we find for the ground state when $l = 0$, the shift of energy

$$\Delta\mathcal{E}_2 = \left(\frac{\lambda^2}{4\mu}+\frac{\gamma^2}{M}\right)\frac{H^2}{3}\langle\xi^2\rangle, \tag{128}$$

where $\langle\xi^2\rangle = \langle 0 \mid \xi^2 \mid 0\rangle$, and $\mid 0\rangle$ means the wave function of the ground s-state. We took into account that the first and the third terms in Eq. (127) equal zero because $L_3 \mid 0\rangle = 0$. Here we ignore the spin interactions of a baryon with the external magnetic field and therefore only diamagnetic polarizability can be defined from Eq. (128). Using the definition of the relative coordinate $\mathbf{r}_b = b\xi$ and comparing Eq. (128) with Eq. (155) gives the diamagnetic polarizability of a baryon

$$\beta^{dia} = -\left(\frac{\lambda^2}{4\mu}+\frac{\gamma^2}{M}\right)\frac{2}{3b^2}\langle\mathbf{r}_b^2\rangle. \tag{129}$$

As required, the diamagnetic polarizability is negative and Eq. (129) is like the Langevin formula for the magnetic susceptibility of atoms. The similar expression was derived in [36] for mesons. Taking into account Eqs. (92), (109), expression (129) is rewritten as

$$\beta^{dia} = -\left(\frac{e^2}{4}+q^2\right)\frac{\langle\mathbf{r}_b^2\rangle}{6M}, \tag{130}$$

where $M = \mu_1^{(0)}+\mu_2^{(0)}+\mu_3^{(0)} = 2\mu_3^{(0)} = 2\sqrt{\sigma}\,[a(n)/3]^{3/4}$. To calculate the value of β^{dia} for nucleons we can use the theoretical magnitude of the mean-squared relative coordinate $\langle \mathbf{r}_b^2 \rangle$ or experimental data for the size of a nucleon. The first way is preferable. Taking into account Eq. (113), the value of M and using the approximate relation $\langle\ \mathbf{r}_b^2 \rangle \simeq \langle \sqrt{\mathbf{r}_b^2} \rangle^2$, Eq. (130) transforms into

$$\beta^{dia} = -\frac{e^2+4q^2}{12\sigma^{3/2}} \left[\frac{a(n)}{3}\right]^{3/4}. \tag{131}$$

For a proton with the more favorable configuration a), when the quark $q^{(3)} = d$ and diquark $(q^{(1)}q^{(2)}) = (uu)$, parameter $q = (5e)/3$ and Eq. (131) (at $\sigma = 0.15$ GeV^2) takes the value

$$\beta_p^{dia} = -8 \times 10^{-4}\ fm^3. \tag{132}$$

This quantity is greater than that found in the nonrelativistic quark model [22]. To calculate β^{para} in the present approach one needs to take into account the interaction of spins of quarks with the magnetic field. For estimation of the total magnetic polarizability of a proton we use the contribution β_Δ^{para} and in accordance with Eqs. (159), (162) we find

$$\bar{\beta}_p = (5 \pm 3) \times 10^{-4}\ fm^3. \tag{133}$$

This quantity is close (within two standard deviations) to the experimental value [52] which was extracted from the measurements of the cross sections of the Compton scattering on hydrogen:

$$\bar{\beta}_p^{\exp} = (2.9 \mp 0.7 \mp 0.8) \times 10^{-4}\ fm^3. \tag{134}$$

The value (133) is also in agreement with quantity (167) found in other schemes. The value $\bar{\beta}_p$ is small due to the partial cancellation of the positive paramagnetic polarizability $\bar{\beta}_p^{para}$ with the negative diamagnetic polarizability $\bar{\beta}_p^{dia}$. The theoretical evaluation, interpretation and experimental extraction of the total magnetic polarizability $\bar{\beta}_p$ is difficult because it is small. That is why this quantity is model dependent in different schemes. The approach considered allows us to improve the accuracy by using the perturbation in the spin interaction. The next step is to take into account such corrections.

For a neutron with the configuration a) where the quark $q^{(3)} = u$ and diquark $(q^{(1)}q^{(2)}) = (dd)$, the parameter $q = -(4e)/3$ and Eq. (131) gives

$$\beta_n^{dia} = -5.4 \times 10^{-4}\ fm^3. \tag{135}$$

Using the paramagnetic polarizability $\beta_{\Delta}^{para} = (13 \pm 3) \times 10^{-4}$ fm^3 [57] for a neutron (see Eq. (162)) we get from Eq. (135) the generalized magnetic polarizability of a neutron

$$\bar{\beta}_n = (7.6 \pm 3) \times 10^{-4} \ fm^3 \tag{136}$$

which is in agreement within two standard deviations with the experimental quantity [52]

$$\bar{\beta}_n^{\rm exp} = (3.2 \mp 1.5 \mp 2.0) \times 10^{-4} \ fm^3.$$

According to Eqs. (120), (122), (133), (136) the sum of nucleon polarizabilities in our approach are given by

$$\bar{\alpha}_p + \bar{\beta}_p = (15 \pm 3) \times 10^{-4} \ fm^3,$$
$$\bar{\alpha}_n + \bar{\beta}_n = (11.8 \pm 3) \times 10^{-4} \ fm^3. \tag{137}$$

Values (137) are close to the reliable results found on the basis sum rule (166). As pion degrees of freedom play a very important role in the nucleon electric polarizability, one needs to describe pions in this scheme. In the approach considered here the accuracy of the values of electromagnetic polarizabilities can be improved by taking into consideration spin interactions as perturbations in accordance with Eq. (87).

10 Cluster Expansion

In this paragraph we go into Euclidean formalism and follow closely the works [16,17,61,66]. Some important characteristics of strong interactions depend on the Wilson integral averaged with certain weights. So, spin-spin and spin-orbit quark interactions can be expressed through the Wilson loop. It is difficult to calculate the Wilson loop integral in the general case of arbitrary contours due to the nonlinear character of gluonic fields. However, it is possible to use the powerful method of cluster expansion [64]. To use this method it is necessary to accept the stochastic nature of the QCD vacuum [16]. Lattice data confirm this conception [29, 61] and give the correlation length $T_g \simeq 0.2 \ fm$.

The method of field correlators allows us to evaluate approximately the Wilson loop using a cumulant expansion. MFC takes into account both perturbative and nonperturbative interactions of gluons and therefore it is applicable for small and large distances.

The Wilson integral is expressed via gauge-invariant and Lorenz covariant correlators of strengths of gluonic fields. To make this expansion one needs to use the non-Abelian Stokes theorem (25).

Let us introduce the generating functional for field correlators of gluonic fields (see [65]) as follows

$$Z[J] = \langle \exp\Big(ig \int dx J_{\mu\nu}(x) F_{\mu\nu}(x, x_0) \Big) \rangle, \tag{138}$$

where $F_{\mu\nu}(x, x_0) = \Phi(x_0, x) F_{\mu\nu}(x) \Phi(x, x_0)$. Using the Taylor expansion in $J_{\mu\nu}$ we arrive at

$$Z[J] = 1 + \sum_{n=1}^{\infty} \frac{(ig)^n}{n!} \int dx_n \cdots \int dx_n J_{\mu_1\nu_1}(x_1) \cdots J_{\mu_n\nu_n}(x_n)$$
$$\times \langle F_{\mu_1\nu_1}(x_1, x_0) \cdots F_{\mu_n\nu_n}(x_n, x_0) \rangle, \tag{139}$$

so that the field correlators (averaged strength tensors over gluonic fields) are given by

$$\langle F_{\mu_1\nu_1}(x_1, x_0) \cdots F_{\mu_n\nu_n}(x_n, x_0) \rangle = \frac{(ig)^{-n} \delta^n Z[J]}{\delta J_{\mu_1\nu_1}(x_1) \cdots \delta J_{\mu_n\nu_n}(x_n)} \,|_{J=0} \,. \tag{140}$$

It is convenient instead of expressions (140) to use irreducible correlators - cumulants [64] in accordance with relationship

$$\langle\langle F_{\mu_1\nu_1}(x_1, x_0) \cdots F_{\mu_n\nu_n}(x_n, x_0) \rangle\rangle = \frac{(ig)^{-n} \delta^n \ln Z[J]}{\delta J_{\mu_1\nu_1}(x_1) \cdots \delta J_{\mu_n\nu_n}(x_n)} \,|_{J=0} \,. \tag{141}$$

From Eq. (141) we come to the expansion of the generating functional $Z[J]$ via cumulants:

$$Z[J] = \exp\Big(1 + \sum_{n=1}^{\infty} \frac{(ig)^n}{n!} \int dx_1 \cdots \int dx_n J_{\mu_1\nu_1}(x_1) \cdots J_{\mu_n\nu_n}(x_n)$$
$$\times \langle\langle F_{\mu_1\nu_1}(x_1, x_0) \cdots F_{\mu_n\nu_n}(x_n, x_0) \rangle\rangle \Big). \tag{142}$$

Making variations of Eqs. (139) and (142) and setting $J_{\mu\nu} = 0$ one arrives at the first three relations [64] (we omit here argument x_0 implaying $F_{\mu\nu}(x, x_0) = F_{\mu\nu}(x)$):

$$\langle\langle F_{\mu\nu}(x) \rangle\rangle = \langle F_{\mu\nu}(x) \rangle,$$

$$\langle\langle F_{\mu_1\nu_1}(x_1)F_{\mu_2\nu_2}(x_2)\rangle\rangle = \langle F_{\mu_1\nu_1}(x_1)F_{\mu_2\nu_2}(x_2)\rangle - \langle F_{\mu_1\nu_1}(x_1)\rangle\langle F_{\mu_2\nu_2}(x_2)\rangle,$$
$$\langle\langle F_{\mu_1\nu_1}(x_1)F_{\mu_2\nu_2}(x_2)F_{\mu_3\nu_3}(x_3)\rangle\rangle = \langle F_{\mu_1\nu_1}(x_1)F_{\mu_2\nu_2}(x_2)F_{\mu_3\nu_3}(x_3)\rangle$$
$$-\langle F_{\mu_1\nu_1}(x_1)F_{\mu_2\nu_2}(x_2)\rangle\langle F_{\mu_3\nu_3}(x_3)\rangle - \langle F_{\mu_1\nu_1}(x_1)\rangle\langle F_{\mu_2\nu_2}(x_2)F_{\mu_3\nu_3}(x_3)\rangle$$
$$-\langle F_{\mu_1\nu_1}(x_1)F_{\mu_3\nu_3}(x_3)\rangle\langle F_{\mu_2\nu_2}(x_2)\rangle + \langle F_{\mu_1\nu_1}(x_1)\rangle\langle F_{\mu_2\nu_2}(x_2)\rangle\langle F_{\mu_3\nu_3}(x_3)\rangle$$
$$+\langle F_{\mu_1\nu_1}(x_1)\rangle\langle F_{\mu_3\nu_3}(x_3)\rangle\langle F_{\mu_2\nu_2}(x_2)\rangle. \tag{143}$$

It should be noted that there are two types of ordering in cumulant expansions [17], [64]. According to the definition of [17], there are no terms in Eq. (143) like

$$\langle F_{\mu_1\nu_1}(x_1)F_{\mu_3\nu_3}(x_3)\rangle\langle F_{\mu_2\nu_2}(x_2)\rangle,\ \langle F_{\mu_1\nu_1}(x_1)\rangle\langle F_{\mu_3\nu_3}(x_3)\rangle\langle F_{\mu_2\nu_2}(x_2)\rangle,$$

which violate path-ordering prescription. For the Gaussian approximation when we retain only the bilocal cumulant both definitions coincide. The average value $\langle F_{\mu\nu}(x)\rangle$ should vanish, because in the stochastic vacuum there is no the definite direction. Otherwise the Lorentz invariance will be broken by vacuum. Applying the cumulant expansion (142) to the average Wilson integral (24) one arrives at

$$\langle W(C)\rangle = \frac{1}{N_C} tr P \exp\Big(\sum_{n=1}^{\infty}\frac{(ig)^n}{n!}\int_\Sigma d\sigma_{\mu_1\nu_1}(x_1)\cdots\int_\Sigma d\sigma_{\mu_n\nu_n}(x_n)$$
$$\times\langle\langle F_{\mu_1\nu_1}(x_1,x_0)\cdots F_{\mu_n\nu_n}(x_n,x_0)\rangle\rangle\Big), \tag{144}$$

where P an ordering operator on the coordinate x of the matrix $F_{\mu\nu}(x,x_0)$, and we imply averaging on gluonic fields with the standard weight (see (5)).

The expansion (144) makes sense when it converges. As we assume that Eq. (144) is valid for any g (even for large distances), cumulant expansion is nonperturbative. Lattice data showed [29, 66] that the correlators decrease when the distance between two points increases and the expansion (144) is justified. It is known that in a Gaussian random process, the higher cumulants vanish, and only the quadratic cumulant does not equal zero. With good accuracy the QCD vacuum can be considered as a stochastic ensemble of gluonic fields [66]. This means that the main contribution to the Wilson loop expansion (144) comes from the bilocal cumulant

$$\langle\langle F_{\mu\nu}(x,x_0)F_{\alpha\beta}(y,x_0)\rangle\rangle = \langle\langle F_{\mu\nu}(x)\Phi(x,y)F_{\alpha\beta}(y)\Phi(y,x)\rangle\rangle. \tag{145}$$

Higher cumulants in Eq. (144) give small corrections with $\sqrt{\langle (F^a_{\mu\nu})^2\rangle} T_g^2 \ll 1$ [16, 17], and can be neglected (Gaussian approximation). The Gaussian dominance was supported by lattice calculations and the hadron phenomenology. Thus we come to the Gaussian stochastic model of the QCD vacuum [29, 66]. The fundamental input in such a model is the Gaussian correlator. But there is a problem; what kind of field configurations (classical or quantum) play the dominant role to form the definite behavior of field correlators.

However, there will be the dependence of Wilson loop integral (144) on the point x_0 and the shape of the surface Σ. We recall that the point x_0 and contour C belong to the surface Σ. The total expression (144) with higher cumulants does not depend on the point x_0 and contour C. To reduce the dependence of a cumulant on the Σ, one should choose the surface with the minimal area [17]. Then the area law of the Wilson integral with large loops (the size $R \simeq 1\ fm$) takes place [17]. To get rid on the dependence of the Wilson loop on x_0 one should have the condition [17]

$$\mid x^{(i)} - x^{(j)} \mid \ll \mid x^{(i)} - x_0 \mid, \quad \mid x^{(j)} - x_0 \mid .$$

Thus, at small correlation length $T_g \ll \mid x^{(i)} - x_0 \mid$ one can neglect the dependence on x_0. In the Gaussian approximation the Wilson integral (144) becomes

$$\langle W(C)\rangle = \frac{1}{N_C} tr P \exp\Big(-\frac{g^2}{2}\int_{\Sigma_{min}} d\sigma_{\mu\nu}(x)\int_{\Sigma_{min}} d\sigma_{\alpha\beta}(y) \times \langle\langle F_{\mu\nu}(x,x_0)F_{\alpha\beta}(y,x_0)\rangle\rangle\Big), \tag{146}$$

where we imply that $x = x(\xi)$, $y = y(\xi)$. Thus, for a size of the Wilson loop $R \gg T_g$ the bilocal cumulant (145) entering Eq. (146) does not depend on x_0. As a result it can be expressed via two scalar functions D_1 and D_2 [16,17]:

$$\frac{g^2}{2}\langle\langle F_{\mu\nu}(x)\Phi(x,y)F_{\alpha\beta}(y)\Phi(y,x)\rangle\rangle = I_{N_C}\Big\{(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})D\,(x-y) + \frac{1}{2}\Big[\frac{\partial}{\partial x_\mu}\Big((x-y)_\alpha\delta_{\nu\beta} - (x-y)_\beta\delta_{\nu\alpha}\Big) + \frac{\partial}{\partial x_\nu}\Big((x-y)_\beta\delta_{\mu\alpha} - (x-y)_\alpha\delta_{\mu\beta}\Big)\Big]D_1\,(x-y)\Big\}, \tag{147}$$

where I_{N_C} is a unit matrix $N_C \times N_C$, and functions D and D_1 are invariants of the renormalization group. As the cumulant (147) is proportional to the unit matrix I_{N_C}, the ordering operator P can be omitted in Eq. (146). Substituting Eq. (147) into Eq. (146) one arrives at Eg. (29) with the string tension [17]:

$$\sigma = \frac{1}{2} \int d^2 x D(x). \tag{148}$$

So, σ is an integral characteristic of the Gaussian correlator. Only function D enters Eq. (148) defining the string tension but both functions D and D_1 give the contribution to the perimeter term $\gamma L(C)$ in Eq. (31) [17]. Higher cumulants add small corrections to the string tension (148). The functions D, D_1 in the nonperturbative regime were derived in lattice experiments [29,66] and are given by

$$D(x) = D(0) \exp\Bigl(-\frac{\mid x \mid}{T_g}\Bigr), \quad D_1(x) = D_1(0) \exp\Bigl(-\frac{\mid x \mid}{T_g}\Bigr), \tag{149}$$

where $D(0) = 0.073\ GeV^4$, $D_1(0) = (1/3)D(0)$, $T_g = 0.2 \div 0.3\ fm$. Values $D(0), D_1(0)$ can be connected with the gluonic condensate [4]

$$\frac{\alpha_s}{\pi} \langle \Bigl(F^a_{\mu\nu}(0)\Bigr)^2 \rangle \simeq 0.012\ GeV^4, \tag{150}$$

so that $D(0) \simeq 2\alpha_s \langle \Bigl(F^a_{\mu\nu}(0)\Bigr)^2 \rangle$. Thus, MFC is a development of the QCD sum rule method. In the perturbative regime at small distances, the contribution from gluonic exchange (giving the Coulomb potential) to the $D(x), D_1(x)$ is of $O(g^2)$ [17]

$$D(x) = 0, \quad D_1(x) = \frac{16\alpha_s}{3\pi x^4}. \tag{151}$$

The colour Coulomb interaction is dominant below 0.3 fm. This region is important for bottomonium ($\bar{b}b$ state) and charmonium ($\bar{c}c$ state) where the sizes of ground states are around 0.2 fm and 0.4 fm, respectively. For $\bar{c}c$ and $\bar{s}s$ systems both perturbative and nonperturbative interactions should be taken into account, and only two D, D_1 functions can be used. It should be noticed that nonperturbative sector influences the perturbative part and vice versa. So, two functions, D, D_1 can be represented as [17,66]

$$D(x) = D^{Pert}(x) + D^{NP}(x), \quad D_1(x) = D_1^{Pert}(x) + D_1^{NP}(x). \tag{152}$$

At $x^2 \to 0$ the perturbative part of the function D has the form $D^{Pert}(x) \sim x^{-\alpha}$, $\alpha > 0$. The nonperturbative part is normalized by the gluon condensate

$$D^{NP}(0) + D_1^{NP}(0) = (1/24N_C)\langle\left(F^a_{\mu\nu}(0)\right)^2\rangle = (\pi^2/6N_C)(0.14 \pm 0.02)\ GeV^4.$$

The value of the gluon condensate was taken from the lattice data.

The strong nonperturbative chromoelectromagnetic fields of the QCD vacuum cause nonperturbative shift of the energy density of the vacuum due to the scale anomaly [4]:

$$\epsilon = \frac{\beta(\alpha_s)}{16\alpha_s}\langle(F_{\mu\nu}(0)^a)^2\rangle. \tag{153}$$

We know that the Gell-Mann - Low function, $\beta(\alpha_s) < 0$ due to asymptotic freedom at small α_s. So, the nonperturbative shift of the vacuum energy density is energetically favorable.

Because only function $D(x)$ contributes to the string tension (148), it is responsible for a condensation of monopoles in accordance with the 't Hooft and Mandelstam scenario of confinement. The lattice calculations gave the width of the string $l = 0.4 \div 0.5\ fm$, so that the correlation length $T_g \simeq (1/2)l$. Thus, the radius of the string is close to T_g. Therefore another interpretation of T_g is the thickness of the confining string. In the case $T_g \to 0$ we have pure string limit. The reader can find more about MFC in [17,66].

11 Conclusion

The QCD string theory allows us to obtain the effective Lagrangians for mesons and baryons and estimate the mean squared radii and electromagnetic polarizabilities of hadrons. These quantities were derived as functions of the string tension which is a fundamental variable in this approach. It is not difficult to calculate the electromagnetic polarizabilities of excited states of hadrons using this approach. For that we should take the quantum numbers $n_r = 1$, $l = 0$ $(n = 2)$ and evaluate the mean relative coordinate in accordance with Eq. (113). Then Eqs. (118), (130) give the required polarizabilities. To have more precise values of the hadron electromagnetic characteristics one needs to take into account spin corrections. Especially it is important for light pseudoscalar mesons (π, K mesons). In principle it is possible to receive the spin-orbit and spin-spin interactions using the general expressions (23), (83). The model of baryons as a quark-diquark system is very similar to the approach for mesons. The mean size and electromagnetic polarizabilities of a proton and neutron are in reasonable agreement with the experimental data. The more favorable combination for a diquark is (uu) for a proton and (dd) for a neutron. In this case theoretical values for electromagnetic polarizabilities are close to experimental data.

It should be noted that the quark-diquark ansatz used here generates an electric dipole moment, and a result, there is a contribution to the energy linear in the electric field which is like the linear Stark effect. This requires the deeper understanding the properties of a nucleon under the symmetry transformations (for example parity).

Spin interactions of quarks treated as a perturbation were neglected here but we took them into account by using the paramagnetic polarizability of a nucleon due to the Δ contribution. As an approximation it is justified because in Isgur-Karl model of baryons [58] spin-orbit splitting are much smaller than expected from one-gluon-exchange matrix elements (spin forces were also discussed in [59]). It was also shown [60,61] that the contributions from the Coulomb and spin-spin interactions cancel each other. Nevertheless all parameters should be taken from the approach and then compared with empirical data. Therefore one needs to calculate the paramagnetic polarizability of a nucleon in our approach.

Implying small spin-orbit forces in baryons we come to K^* and quark-diquark system correspondence (see [36]). The present approach can be applied for studying any baryon (see also [60]).

The Nambu-Jona-Lasinio (NJL) model [13,14] having a basis in the framework of QCD [12] describes the chiral symmetry breaking but not the confinement of quarks [17,15]. Besides this model has free parameters and the calculated polarizabilities of mesons [62] are parameter dependent.

The instanton vacuum theory developed in [11] does not give the confinement of quarks phenomenon [17]. This theory is like the NJL model [15] and takes into account only the chiral symmetry breaking. Therefore the calculation of the meson electromagnetic polarizabilities on the basis of the IVT gave the similar result [63] as in the NJL model.

The electromagnetic polarizabilities of nucleons found are close to the values calculated in the framework of the dispersion sum rule [71] and the chiral perturbation theory (CHPT) [72].

To improve the accuracy of calculations of electromagnetic polarizabilities of nucleons one should take into account pion degrees of freedom because the pion cloud contributes substantially to electromagnetic properties (see Appendix).

All this shows that the theoretical evaluation of the charge radii and the magnetic polarizabilities of hadrons is possible on the basis of a good description the chiral symmetry breaking and confinement of quarks in the framework of the QCD string theory but with some approximations and model assumptions. Naturally this theory was derived using the non-perturbative QCD, i.e. first principals of QCD.

12 Appendix

In this Appendix we define nucleon electromagnetic polarizabilities in different approaches. The static electric α and magnetic β polarizabilities of a hadron characterize its internal structure. These quantities are low-energy characteristics of a hadron and therefore they are defined in the non-perturbative regime of QCD. In external electromagnetic fields the hadron is deformed and induces dipole electric and magnetic moments

$$\mathbf{D} = \alpha \mathbf{E}, \quad \mathbf{M} = \beta \mathbf{H}. \tag{154}$$

The acquired dipole moments (154) of a hadron give the contribution to the potential energy as follows

$$U(\alpha, \beta) = -\frac{1}{2}\alpha E^2 - \frac{1}{2}\beta H^2. \tag{155}$$

We use Gaussian units here. The cross section of scattering of a photon on hadrons (Compton scattering) depends on the electromagnetic polarizabilities due to Eq. (155). Thus, low-energy Compton scattering can give important information about the internal structure of hadrons. Below we discuss the nucleon electromagnetic polarizabilities. The Compton amplitude for scattering a photon on a nucleon at low energies in the nucleon rest frame (the laboratory system) is given by [22]

$$f(\gamma N \to \gamma N) = f_B + \omega\omega'\bar{\alpha}(\epsilon\epsilon') + \bar{\beta}(\mathbf{k} \times \epsilon)(\mathbf{k}' \times \epsilon') + O(\omega^3), \tag{156}$$

where $(\mathbf{k}, i\omega)$, ϵ and $(\mathbf{k}', i\omega')$, ϵ' are four momenta and polarization vectors of the incoming and outgoing photon, respectively. The electromagnetic polarizabilities $\bar{\alpha}$, $\bar{\beta}$ occurring Eq. (156) are so-called Compton polarizabilities.

Below we will discuss the interrelation between static and Compton polarizabilities. The f_B is the Born amplitude which is due to the nucleon electric charge eZ and anomalous magnetic moment κ of a nucleon. The Born amplitude f_B includes the Thomson amplitude f_T which is energy-independent and satisfies the well-known low-energy theorem:

$$f_T = -\frac{e^2 Z^2}{4\pi M_N}(\epsilon\epsilon'), \tag{157}$$

where M_N is the nucleon mass. The Thomson scattering amplitude (157) corresponds to a pointlike particle and the total amplitude (156) includes structure parameters - electromagnetic polarizabilities $\bar{\alpha}$, $\bar{\beta}$.

In the nonrelativistic approximation the generalized electric and magnetic polarizabilities which are extracted from measurements of the Compton scattering cross sections are given by [22]

$$\bar{\alpha} = \alpha + \Delta\alpha,$$
$$\alpha = 2\sum_{n\neq N}\frac{\mid\langle n\mid D_z\mid N\rangle\mid^2}{\mathcal{E}_n - \mathcal{E}_0}. \tag{158}$$
$$\bar{\beta} = \beta^{para} + \beta^{dia},$$
$$\beta^{para} = 2\sum_{n\neq N}\frac{\mid\langle n\mid M_z\mid N\rangle\mid^2}{\mathcal{E}_n - \mathcal{E}_0}. \tag{159}$$

where $\mid n\rangle$ is the excited state of a hadron, D_z and M_z are the third projections of the electric and magnetic dipole operators, respectively. The excited state $\mid n\rangle$ includes the meson-baryon intermediate states. In nonrelativistic calculations the $\Delta\alpha$ is due to form factors and depends on the electric charge radius. Eqs. (158), (159) are similar to the usual quantum-mechanical formulas. Eq. (158) was also confirmed by the quasiclassical calculations of pion polarizability in the framework of the instanton vacuum theory [63]. In the completely relativistic approach the physical meaning of $\bar{\alpha}$ is more complicated [22]. In [22] the correction $\Delta\alpha$ is given by

$$\Delta\alpha = \frac{e^2 r_E^2}{3M_N} + \frac{e^2\left(\kappa^2+1\right)}{4M_N^3}, \tag{160}$$

where r_E is the electric radius and the magnetic moment of the hadron is given by $\mu = (e\kappa)/(2M_N)$.

For the proton and neutron, using the experimental values of electric radii and magnetic moments, the term (160) becomes [22]

$$\Delta\alpha_p = (4.5 \pm 0.1) \times 10^{-4}\ fm^3, \quad \Delta\alpha_n = 0.62 \times 10^{-4}\ fm^3 \tag{161}$$

The strong magnetic $N\Delta$ transition gives rise to a positive part of the magnetic polarizabilities. Therefore the main contribution to the paramagnetic polarizability of a nucleon is due to $\Delta(1232)$ excitation [57] and is given by

$$\beta_\Delta^{para} = (13 \pm 3) \times 10^{-4}\ fm^3. \tag{162}$$

The intermediate state Δ is an isovector excitation and contributes to the proton and neutron so that $\beta_\Delta^p = \beta_\Delta^n$. The interesting feature of the experimental data is that for the proton and the neutron the approximate equality $\bar{\alpha}_p + \bar{\beta}_p \simeq \bar{\alpha}_n + \bar{\beta}_n$, and inequality $\bar{\alpha}_p, \bar{\alpha}_n \gg \bar{\beta}_p, \bar{\beta}_n$ are valid. The last inequality means that both a proton and a neutron behave like electric dipoles. In addition, the large positive paramagnetic polarizability of nucleons should be canceled by the diamagnetic contribution. The sum of electromagnetic polarizabilities can be connected with the total photoabsorption cross section [67,68,22]. Indeed, from Eq. (156) the spin-independent amplitude of the Compton scattering at low energies and zero angle and $\epsilon = \epsilon'$ is given by

$$f_{\gamma\to\gamma'}(\omega, 0) = -\frac{e^2 Z^2}{4\pi M_N} + (\bar{\alpha} + \bar{\beta})\omega^2 + O(\omega^3). \tag{163}$$

Using the optical theorem [22]

$$Im f_{\gamma\to\gamma'}(\omega, 0) = \frac{\omega}{4\pi}\sigma_{tot}(\omega), \tag{164}$$

and once-subtracted forward dispersion relation for amplitude (163), one finds the relation [67,68]

$$(\bar{\alpha} + \bar{\beta}) = \frac{1}{2\pi^2}\int_{\omega_{thr}}^{\infty} \frac{d\omega}{\omega^2}\sigma_{tot}(\omega), \tag{165}$$

where $\omega = (s - M_N^2)/(2M_N)$ is the photon energy in the laboratory system and $\omega_{thr} = m_\pi(1 + m_\pi/(2M_N))$ is the pion production threshold ($m_\pi = 139.57\ MeV$ is the pion mass). Eq. (165) occurs also for spin-0 particles. The strict equality (165) allows us to extract the sum of electromagnetic polarizabilities from the experimentally measured total photoabsorption cross section. The result is [69,70]

$$(\bar{\alpha}_p + \bar{\beta}_p) = (14.3 \pm 0.5) \times 10^{-4}\ fm^3,$$

$$(\bar{\alpha}_n + \bar{\beta}_n) = (15.8 \pm 0.5) \times 10^{-4}\ fm^3. \tag{166}$$

The most difficult task is to find electromagnetic polarizabilities separately (see [22]).

It follows from Eq. (165) that at energies $\omega \simeq m_\pi$ the sum of polarizabilities behave as $O(m_\pi^{-1})$ because the total photoabsorbtion cross section $\sigma_{tot}(\omega)$ is finite at $\omega \simeq m_\pi \to 0$ [71]. In the chiral limit, $m_\pi \to 0$, both polarizabilities $\bar{\alpha}$, $\bar{\beta}$ diverge as $O(m_\pi^{-1})$. This was confirmed in the framework of the chiral perturbation theory (CHPT) [72]. The same behavior of pion polarizabilities was found on the basis of quasiclassical calculations [63] in the framework of the instanton vacuum theory. Thus, the leading terms of electromagnetic polarizabilities are determined by CSB.

The contribution of low-lying intermediate states, where soft pions play an important role, into the integral of Eq. (165) was estimated in [71]:

$$\bar{\alpha} = \frac{5}{12m_\pi}\left(\frac{eg_{\pi N}\sqrt{2}}{8\pi M_N}\right)^2 \simeq 13.6 \times 10^{-4}\ fm^3,$$

$$\bar{\beta} = \frac{1}{24m_\pi}\left(\frac{eg_{\pi N}\sqrt{2}}{8\pi M_N}\right)^2 \simeq 1.4 \times 10^{-4}\ fm^3, \tag{167}$$

where the strong pion-nucleon coupling constant $g_{\pi N} = 13.4$ is connected with the charged pion decay constant $f_\pi = 93\ MeV$ by the Goldberger-Treiman relation $g_{\pi N}/M_N = g_A/f_\pi$, where g_A is the axial-vector coupling constant. The values (167) found for the leading contribution in the chiral limit, and therefore $\bar{\alpha}$, $\bar{\beta}$ are the same for the proton and neutron. The next corrections $O(\omega/M_N)$ to polarizabilities, due to the nucleon recoil, are different for a proton and neutron [71]. The result (167) is in agreement with CHPT calculations [72].

The nucleon polarizabilities were also evaluated on the basis of nucleon soliton models in [73-79] .

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